

Development of computational tool for modeling parallel-scale and “Z chart” nomograms - Calculation of the fundamental frequency of steel-concrete composite beams

Desenvolvimento de ferramenta computacional para modelagem de nomogramas tipos escalas paralelas e “Z” - Cálculo da frequência fundamental de vigas mistas de aço-concreto

Desarrollo de herramienta computacional para modelaje de nomogramas tipo escalas paralelas y “Z” – Cálculo de la frecuencia fundamental de vigas mezcladas de acero-hormigón

Développement de l’outil numérique pour la modélisation des nomogrammes types échelles parallèles et «Z» - calcul de la fréquence fondamentale de poutres développées dans une combinaison d’acier et de béton

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Abstract

Steel-concrete composite beams have become increasingly common in engineering works. These structures require careful attention to the occurrence of excessive vibrations, mainly due to the low weight, possibility of long spans and low damping capacity. This work presents the development of a computer program capable of generating nomograms for determining the fundamental frequency of steel-concrete composite beams. Nomograms are rarely used in engineering nowadays; however, they can be a very useful tool. Furthermore, the tool is part of a larger application for designing steel-concrete composite beams, so it integrates a more comprehensive platform. The computer program developed can generate parallel-scale nomograms and Z chart nomograms in order to check the serviceability limit state of vibrations in composite beams. The accuracy of the nomograms evaluated through a case study has proved the tool to be intuitive, useful and efficient, as the nomograms presented accurate results.

Keywords: Composite beams. Nomogram. Fundamental frequency.

Resumo

Vigas mistas de aço e concreto têm se tornado cada vez mais comuns nas obras de engenharia. Em virtude do baixo peso, possibilidade de grandes vãos e baixa capacidade de amortecimento do aço, estruturas desse tipo exigem que sejam tomados cuidados com relação ao surgimento de vibrações excessivas. O presente trabalho trata do desenvolvimento de um programa computacional capaz de gerar nomogramas para o cálculo da frequência fundamental de vigas mistas. A utilização de nomogramas na engenharia é pouco explorada atualmente, apesar de se mostrar uma ferramenta bastante prática. Além disso, tal ferramenta faz parte de um programa para dimensionamento de vigas mistas, formando uma plataforma maior e mais abrangente. Obteve-se como resultado final um programa computacional capaz de gerar nomogramas tipo “W” e tipo “Z” para verificação do estado limite de vibrações de vigas mistas. Foi realizado um estudo de caso para avaliação da precisão dos nomogramas gerados pelo programa, que se mostrou intuitivo, prático e eficiente, pois os nomogramas gerados apresentaram respostas precisas.

Palavras-chave: Vigas mistas. Nomograma. Frequência fundamental.

Resumen

Vigas mezcladas de acero y hormigón están cada día más frecuentes en las obras de ingeniería. Por causa del bajo peso, posibilidad de grandes huecos y baja capacidad de amortiguación del acero, estructuras de este tipo exigen cuidados con relación al surgimiento de vibraciones excesivas. El presente trabajo trata del desarrollo de un programa computacional capaz de generar nomogramas para el cálculo de la frecuencia fundamental de vigas mezcladas. El uso de nomograma en la ingeniería es poco explorado hoy día, aunque sea una herramienta bastante práctica. Además, tal herramienta forma parte de un programa para dimensionamiento de vigas mezcladas, formando una plataforma más grande y más amplia. Como resultado final se obtuvo un programa computacional capaz de generar nomogramas tipo “W” y tipo “Z” para comprobación del estado límite de vibraciones de vigas mezcladas. Se realizó un estudio de caso para evaluación de precisión de los nomogramas generados por el programa, que se mostró intuitivo, práctico y eficiente, porque los nomogramas generados presentaron respuestas precisas.

Palabras-clave: Vigas mezcladas. Nomograma. Frecuencia fundamental.

Résumé

Les poutres développées dans une combinaison d’acier et de béton sont devenues de plus en plus courantes dans les travaux d’ingénierie. Ce phénomène arrive, car les poutres ont du faible poids, de la possibilité de grandes travées et de la faible capacité d’amortissement de l’acier. Telles structures nécessitent des soins particuliers en cas de vibrations excessives. Ce travail aborde le développement d’un programme numérique capable de générer des nomogrammes pour le calcul de la fréquence fondamentale des poutres mixtes. L’utilisation de nomogrammes dans l’ingénierie est aujourd’hui peu explorée, même avec des évidences d’être un outil très pratique. En outre, cet outil est partie d’un programme pour le dimensionnement des poutres mixtes, ce qui développe une plate-forme plus grande et complète. Comme résultat final, on a obtenu un programme numérique capable de générer des nomogrammes type «W» et type «Z» qui vérifient l’état limite des vibration des poutres mixtes. Une étude de cas a été réalisée avec l’objectif d’évaluer la précision des nomogrammes générés par le programme. Le programme s’est montré intuitif, pratique et efficace, car les nomogrammes générés ont présenté des réponses précises.

Mot-Clés: Poutres mixtes. Nomogramme, Frequence fondamentale.

1 Introduction

The assertion that “Civil Engineering has traditionally assumed that the forces acting on its structures are constant in time” (NÓBREGA, 2004, p.10) refers to the static structural analysis. As Sommer (2002) points out, before the industrial revolution the structures in general, built from heavy pieces of wood, had great mass, which provided great damping inherent to the structure.

Over time, the use of lighter and more resistant materials such as steel; the improvement of structural calculation processes and the development of new structural systems have enabled the construction of bolder structures and longer spans. Among these new structural systems, steel-concrete composite beams are being increasingly used in civil construction due to a more efficient utilization of each material property. This is significant “not only because of the resistant capacity but also from a functional and aesthetics point of view” (SILVA JÚNIOR, 2015, p.1). This composite system provides speed in the execution phase, reduction of the total weight of the structure, the possibility of longer spans and slenderer elements, among other positive aspects.

However, these characteristics are directly linked to the high sensitivity of these systems to dynamic demands, especially those from human activities, which can result in excessive vibrations. In this way, verifications regarding the limit state of vibrations in structures of mixed beams have become essential to avoid the malfunction of the structure and the discomfort of its users. Therefore, there is a demand for increasingly specific computer programs capable of performing analyzes and calculations of parameters of a dynamic nature.

An important tool in this work is the nomography, that is, the graphical representation of mathematical relations. These graphs, also known as nomograms, abacuses, or alignment charts, were used “extensively in the 1970s to provide engineers with quick graphing of complicated formulas with practical accuracy” (DOERFLER, 2009). As computers and

calculators became more common, the nomograms were set aside and now look old-fashioned. However, nomograms can simplify the calculation of important parameters in the dynamic structural analysis, such as fundamental frequency, as well as enable a better visualization of the results.

This work tries to show that the potential of nomography can be extended by using computer programming, like Zhang and Kattan (2017) did, who describe how to draw nomograms for outcome predictions in clinical medicine using the programming language R, since “the equation describing the relationship between predictors and outcome is often complex and the computation requires software for practical use” (ZHANG; KATTAN, 2017).

Most applications and researches using nomography can be found in medicine. Flaherman (2014) demonstrates how nomograms can be used for the early identification of neonates on a trajectory for increased weight loss and related morbidities. Karlo *et al.* (2016) present a nomogram for the prediction of indolent renal cortical tumors. Fenghao Sun *et al.* (2017) present a nomogram to predict prognosis post surgery in early-stage non-small cell lung cancer in elderly patients and Rick G. *et al.* (2013) present a validated web-based nomogram for predicting positive surgical margins after breast-conserving surgery as a preoperative tool for clinical decision-making.

Thus, this work presents the development of a computational tool for modeling nomograms that enable the calculation of the fundamental frequency of steel-concrete composite beams. Two types of nomograms will be used: parallel-scale nomograms and “Z” charts. The tool also facilitates the analysis of parameters that can interfere with the frequency values. The tool was developed in conjunction with a program that verifies the ultimate limit state of steel-concrete composite beams. So, it composes an application created for designing composite beams. The application was named “IVM – I Vigas Mistas”.

Besides, an interface was developed for the program, and an algorithm was created for rendering nomograms. At the end of this work, a case study is carried out to evaluate the efficiency and accuracy of the nomograms generated by the program.

2 Calculation of natural frequencies

In the analysis of a linear structure, it is ideal to distinguish between the static and dynamic components of the load. Their total effect is obtained by overlapping the separate effects of each one since, according to Clough and Penzien (2003), static and dynamic analysis methods are fundamentally different.

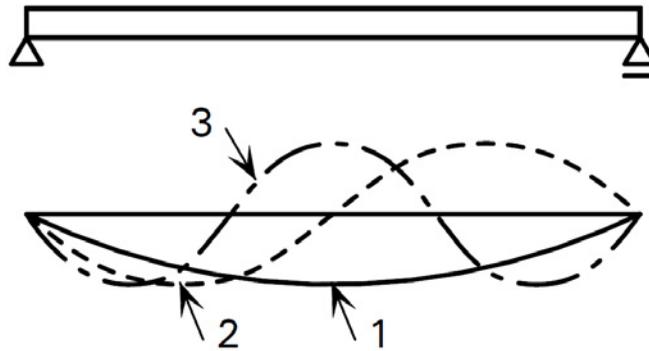
The term “dynamic”, when associated with the study and analysis of stresses in structures, is defined simply as a variable in time. Thus, a dynamic load can be defined as “any load of which its magnitude, direction and/or position varies with time” (CLOUGH; PENZIEN, 2003, p.1, our translation). Likewise, the resulting stresses and deformations are also time-varying or dynamic.

The result of the performance of rhythmic dynamic loads in the structures is, among other effects, the appearance of vibrations, which reach their peak when the loads present variations with a frequency similar to the natural frequencies of the structure, as described by Silva (2007). This effect is known as resonance.

The natural frequencies of a system, or resonant frequencies, are the ones that result in the structure oscillating in free vibration. The frequency is measured in Hz (cycles per second). Each system has a series of natural frequencies, each one corresponding to a mode of vibration. A mode of vibration shows the shape of the system when presenting maximum deflection. The first mode of vibration represents the lowest frequency value, known as fundamental frequency. Figure 1 schematically shows the first three modes (1, 2 and 3) of vibration of a simply-supported beam.

According to Ellingwood (1984), human activities such as running, dancing and jumping cause excitations with frequencies in the range of 2 to 6 Hz, which emphasizes that systems characterized by large spans and reduced weight tend to present fundamental frequency in this same range, making more the effect of resonance in these systems.

Figure 1 - Modes of vibration of a simply supported beam



Source: Hicks *et al.* (2009)

The calculation of natural frequencies is an essential part of any vibration problem, since “until these frequencies are determined, the effects of any external forces on the system cannot be predicted” (HICKS *et al.*, 2009). For instance, a load falling at each interval of 1.00 s will cause a large reaction in a system with a frequency of 1.00 Hz, due to the resonance effect, but no reaction will be caused in a system with a frequency of 0.01 Hz or in a system with a frequency of 100.00 Hz.

The natural frequency f_n of the n th mode of vibration of a beam with uniform cross section can be calculated using a simplification of the equation of motion for a beam in bending, Eq. (1):

$$m(x) \frac{\partial^2 v(x, t)}{\partial t^2} + EI(x) \frac{\partial^4 v(x, t)}{\partial x^4} = p(x, t) \tag{1}$$

Where:

t = Time;

x = Position along the beam;

m = Distributed mass;

$EI(x)$ = Bending rigidity of the beam;

$v(x, t)$ = Transverse displacement of the beam as a function of x and t ;

$p(x, t)$ = Applied force as a function of x and t ;

When the force function $p(x, t)$ is defined as equal to zero, that is, when it is assumed a state of free vibration, then:

$$EI \frac{\partial^4 v(x, t)}{\partial x^4} = -m \frac{\partial^2 v(x, t)}{\partial t^2} \tag{2}$$

By separating the variables, the function $v(x, t)$ can be expressed as:

$$v(x, t) = W(x)T(t) \tag{3}$$

In this case, $W(x)$ represents the displacement of the beam as a function of the variable x . $T(t)$ determines the amplitude of vibration as a function of the variable t . Applying Eq. (3) to Eq. (2) and dividing all equation terms by $mW(x)T(t)$ the result is:

$$\frac{EI}{mW(x)} \frac{d^4 W(x)}{dx^4} = -\frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} \tag{4}$$

Both terms of Eq. (4) can be made equal to a constant, as follows:

$$\frac{EI}{mW(x)} \frac{d^4 W(x)}{dx^4} = a \tag{5}$$

$$-\frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} = a \quad (6)$$

The function $T(t)$ for continuous systems in free vibration is expressed by Clough and Penzien (2003) as:

$$T(t) = A \cos(f_n t) + B \sin(f_n t) \quad (7)$$

A and B are constants that depend on the initial conditions of displacement and velocity. The following expression is obtained when replacing Eq. (7) in (6):

$$a = f_n^2 \quad (8)$$

And when substituting for Eq. (5) the result is:

$$\frac{d^4 W(x)}{dx^4} - \beta^4 W(x) = 0 \quad (9)$$

Where:

$$\beta^4 = \frac{mf_n^2}{EI} \quad (10)$$

Assuming a solution for $W(x)$ in the form:

$$W(x) = Ce^{sx}, \quad (11)$$

where C and s are constant; replacing them in Eq. (8), the result of its derivative with respect to x is:

$$(s^4 - \beta^4)Ce^{sx} = 0 \quad (12)$$

The roots of Eq. (12) are:

$$s_{1,2} = \pm \beta \quad s_{3,4} = \pm i\beta \quad (13)$$

By incorporating these roots separately into Eq. (9) and adding the four resulting terms, the complete solution is obtained:

$$W(x) = C_1 e^{\beta x} + C_2 e^{-\beta x} + C_3 e^{i\beta x} + C_4 e^{-i\beta x} \quad (14)$$

Where C_1, C_2, C_3 and C_4 are constants. Eq. (14) can also be expressed as:

$$W(x) = C_1 \cos(\beta x) + C_2 \sin(\beta x) + C_3 \cosh(\beta x) + C_4 \sinh(\beta x) \quad (15)$$

Assuming a simply-supported beam with span L , the functions $W(x)$ and $W''(x)$ for the position $x=0$ are:

$$W(0) = C_1 \cos(0) + C_2 \sin(0) + C_3 \cosh(0) + C_4 \sinh(0) = 0 \quad (16)$$

$$W''(0) = \beta^2 (-C_1 \cos(0) - C_2 \sin(0) + C_3 \cosh(0) + C_4 \sinh(0)) = 0 \quad (17)$$

From Eq. (16) it is found that $(C_1 + C_3) = 0$ and from Eq. (17) it is found that $(-C_1 + C_3) = 0$. Therefore $(C_1 = C_3) = 0$. Furthermore, for position $x = L$, the expressions became:

$$W(L) = C_2 \sin(\beta L) + C_4 \sinh(\beta L) = 0 \quad (18)$$

$$W''(L) = \beta^2 [-C_2 \sin(\beta L) + C_4 \sinh(\beta L)] \quad (19)$$

Canceling β^2 and adding Eq. (18) and (19), it results in:

$$2C_4 \sinh(\beta L) = 0 \quad (20)$$

Therefore $C_4=0$, since $\sinh(\beta L) \neq 0$. Then, only C_2 is left, so that:

$$W(x) = C_2 \sin(\beta L) \tag{21}$$

When excluding solution $C_2 = 0$, the boundary conditions of the simply-supported beam will only be satisfied if $\sin(\beta L)=0$, which results in:

$$\beta = \frac{n\pi}{L} \quad n = (1, 2, 3 \dots), \tag{22}$$

where corresponds to the vibration mode. Replacing Eq. (22) into (10), it follows that:

$$f_n = n^2 \pi^2 \sqrt{\frac{EI}{mL^4}}, \tag{23}$$

resulting in:

$$f_n = \frac{k_n}{2\pi} \sqrt{\frac{EI}{mL^4}} \tag{24}$$

where:

EI = bending rigidity of the beam [N/m²]

m = effective distributed mass along the span [kg/m]

L = effective span of the beam [m]

k_n = constant that represents the boundary condition to the nth mode of vibration.

Some values of k_n are shown in Table 1, they were determined for different boundary conditions:

Table 1 – Values of k_n

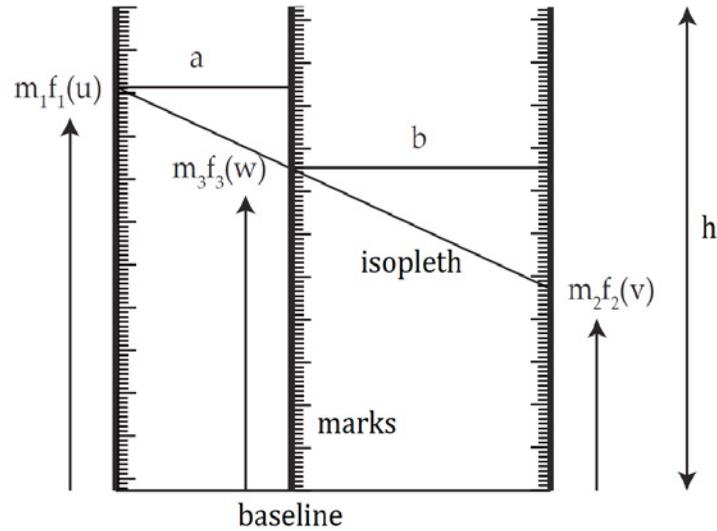
Support Conditions	for nth mode		
	n = 1	n = 2	n = 3
pinned/pinned (“simply-supported”)	π^2	$4\pi^2$	$9\pi^2$
fixed both ends	22,40	61,70	121,00
fixed-free (cantilever)	3,52	22,00	61,70

Source: Hicks *et al.* (2009)

3 Nomography

The basic model of a nomogram is formed by three or more straight or curved scales. Each scale represents the function of a variable that belongs to an equation. For determining one of the variables, a line is drawn through the scales whose values are known, this line is called isopleth. The point where the isopleth crosses the scale of the unknown variable indicates the value of the same. Nomograms consisting of straight scales can be designed from simple geometric relationships; they are the most common type of nomograms and can cover a wide variety of calculations (Figure 2).

Figure 2 – Representation of a parallel-scale nomogram



Source: Adapted from Doerfler (2009)

3.1 Parallel-scale nomogram

In this type of nomogram, the scales are parallel to one another. Each function is plotted according to a corresponding scale factor in order to allow a convenient representation, as defined by Doerfler (2009). The scale factor is equal to the ratio between the height assumed for the scale and the difference between the maximum and minimum values covered by it. The parallel-scale nomogram of the function $f_3(w) = f_1(u) + f_2(v)$ is structured as shown in Fig. 2.

By using the properties of similar triangles, the following equations can be obtained:

$$\frac{m_1 f_1(u) - m_3 f_3(w)}{a} = \frac{m_3 f_3(w) - m_2 f_2(v)}{b} \quad (25)$$

$$m_1 f_1(u) + (a/b)m_2 f_2(v) = (1 + a/b)m_3 f_3(w) \quad (26)$$

The scale factors m_1 , m_2 and m_3 , can be determined by the ratio between the height h adopted for the scale and the difference between the maximum and minimum values, so:

$$m_1 = \frac{h}{f_1(u_{max}) - f_1(u_{min})} \quad (27)$$

$$m_2 = \frac{h}{f_2(v_{max}) - f_2(v_{min})} \quad (28)$$

$$m_3 = \frac{m_1 m_2}{m_1 + m_2} \quad (29)$$

Thus, the values of a and b can be found through the relation:

$$\frac{a}{b} = \frac{m_1}{m_2} \quad (30)$$

The scale factors allow you to set the mark heights on the nomogram scales by converting the values of $f_1(u)$, $f_2(v)$ and $f_3(w)$ to be represented, for corresponding length values. Thus, for the scales of f_1 , f_2 and f_3 respectively, the marks can be determined by:

$$m_1 [f_1(u) - f_1(u_{min})] \quad (31)$$

$$m_2[f_2(v) - f_2(v_{min})] \tag{32}$$

$$m_3[f_3(w) - f_3(w_{min})] \tag{33}$$

Doerfler (2009) also emphasizes that the use of logarithmic scales instead of linear scales expands the use of parallel-scale nomograms to more complicated equations, allowing multiplications to be represented by additions and potentiations to be represented by multiplications.

3.2 “Z” chart nomogram

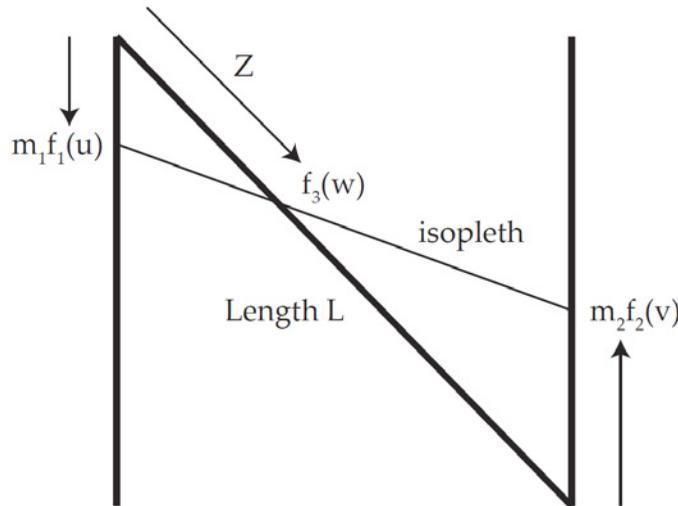
The basic difference between the “Z” chart and the parallel-scale nomograms is the central scale, which is connected to the starting point of the other two scales, as schematized in Fig. 2. In addition, the initial values of the scales must be equal to 0 (zero), so that the mathematical indetermination is attended to when an isopleth is drawn from these values.

The process of designing a “Z” chart nomogram is the same as designing parallel-scale nomogram, however, in order to determine the position of a certain value “N” in the central scale, it is necessary to calculate the length “Z” as indicated in Fig. 3. This length can be found by using Eq. (34):

$$Z = \frac{L * N}{\left(\frac{m_1}{m_2}\right) + N} \tag{34}$$

where L is equal to the total length of the central scale in the unit of desired length.

Figure 3 – Representation of a “Z” chart nomogram



Source: Adapted from Doerfler (2009)

4 Methodology

The program was developed using Object Pascal, which is a programming language derived from Pascal. It supports an object-oriented approach that enabled the development of the current interface for the application. In addition to the libraries and components preset for Object Pascal, OpenGL was also used. It is an OpenGL-based 3D library for Delphi, C++ Builder, and Lazarus. It provides visual components and objects that allow the description and rendering of 2D and 3D scenes. The use of this library was motivated by the ease provided by itself in the implementation of methods for 2D modeling and by the need of a tool capable of rendering nomograms with precision and quality

The algorithm developed for modeling the nomograms was designed so that they present three scales. The scales correspond to the moment of inertia, the fundamental frequency and the free span of the beam, respectively. For each nomogram generated by the program, the value of the mass distributed along the span is fixed, so it is not an input parameter

in the nomogram. The moment of inertia calculated by the program is the moment of inertia of the transformed section, as the concrete area is transformed into an equivalent area of steel, according to NBR 8800: 2008. Therefore, the elastic modulus of steel is the one considered.

At the end of the development of the program a study of a fictitious mixed beam case was carried out to verify the efficiency of the program and the accuracy of the results provided. The analytical calculation of the fundamental frequency of one of the beams of the structure was performed and the result was compared with the program response.

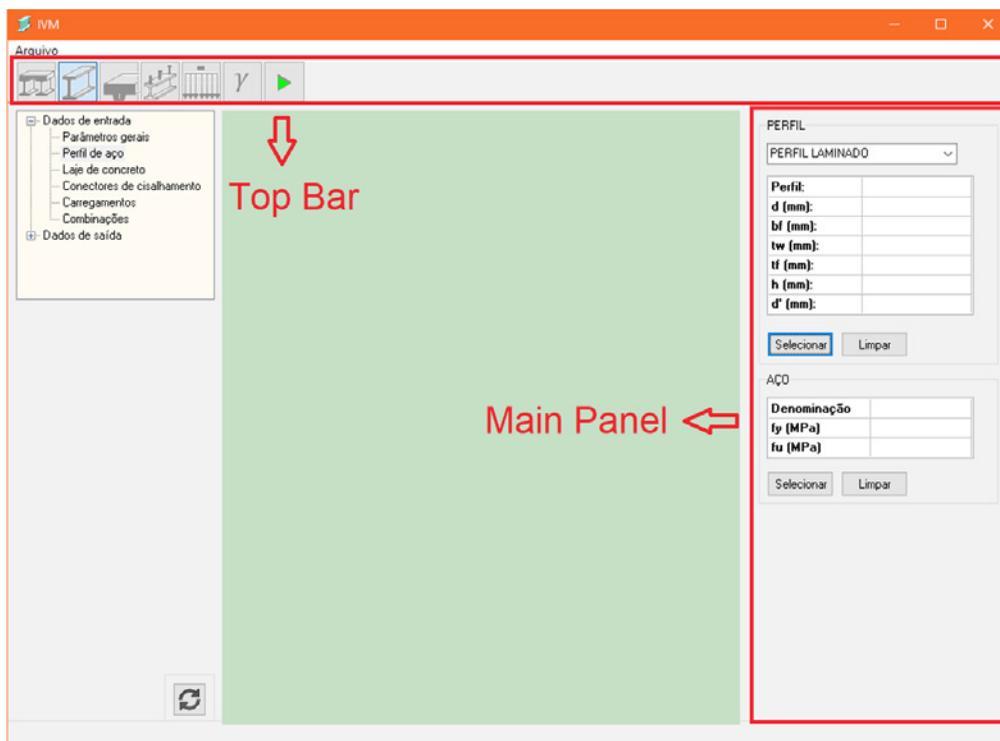
5 The computer program

Figure 4 shows the initial form where the main panels are contained, selected from the top menu. In these panels can be declared the data of the structure and accessed the forms of selection of the structural profile, profile of the connector of shear and steel mold, if the type of slab selected is *steeldeck*.

After all the parameters have been declared, a form presenting the results can be accessed. This form shows a tab for the limit state of vibration, Figure 5. This tab displays the panel in which the nomogram can be generated for calculating the fundamental frequency, an isopleth table is also generated. The table consists of five columns arranged in the following order: isopleth number; distributed mass, which is defined from the frequent combination previously calculated; the free span of the beam; a moment of inertia of the transformed section, and fundamental frequency.

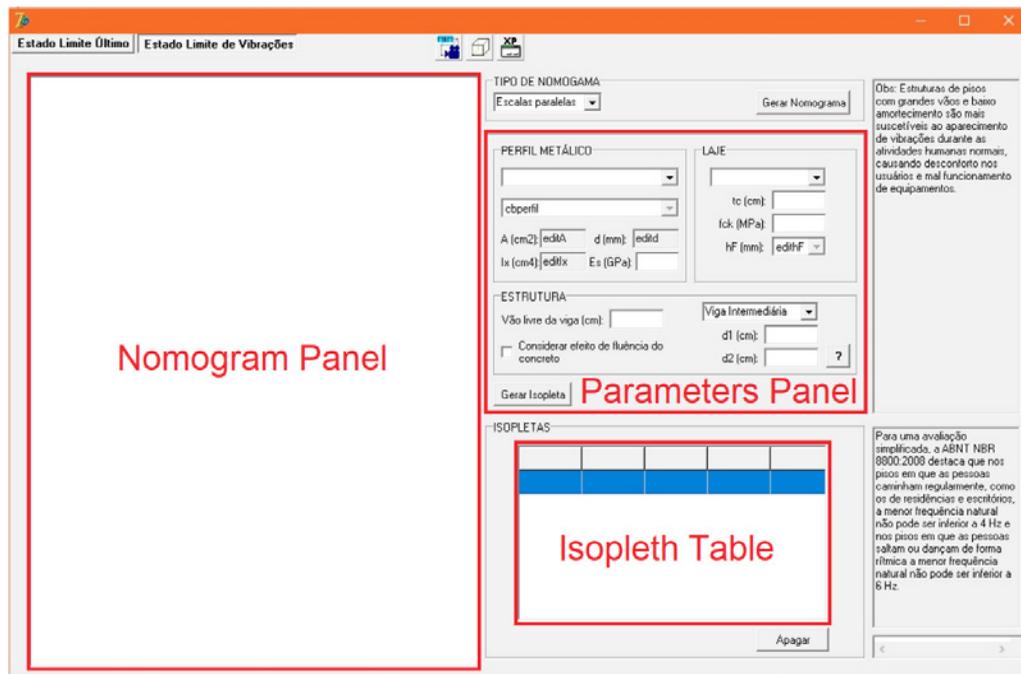
In addition to the nomogram panel and the isopleth frame, the vibration limit check tab contains data inputs for all the parameters required to calculate the moment of inertia of the homogenized section already declared in the main form of the software. These parameters are: denomination of the steel profile, chosen from the database; modulus of elasticity of steel; type of slab; concrete thickness; f_{ck} of the concrete; height of the steel shape, if the selected slab type is *steeldeck*; free of the structure and the consideration or not of the creep effect of the concrete.

Figure 4 – Main form of the software



Source: Elaborated by the authors.

Figure 5 – Tab for the limit state of vibrations

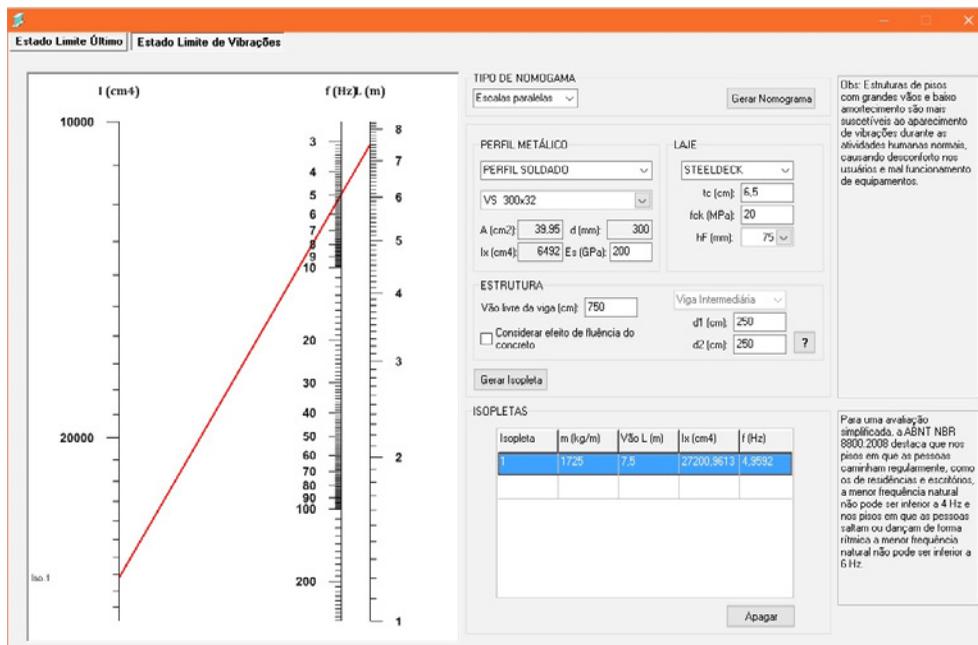


Source: Elaborated by the authors

The widths d_1 and d_2 and the position of the beam can be visualized, but they cannot be altered, as this would result in a significant change in the distributed mass, then the values shown in the nomogram would not be correct.

When the results form is accessed, these input data are associated with the same previously declared values and the nomogram is generated to calculate the fundamental frequency as a function of the moment of inertia and the free span of the beam, and the isopleth number 1, corresponding to the original values of the structure, as can be seen in Fig. 6

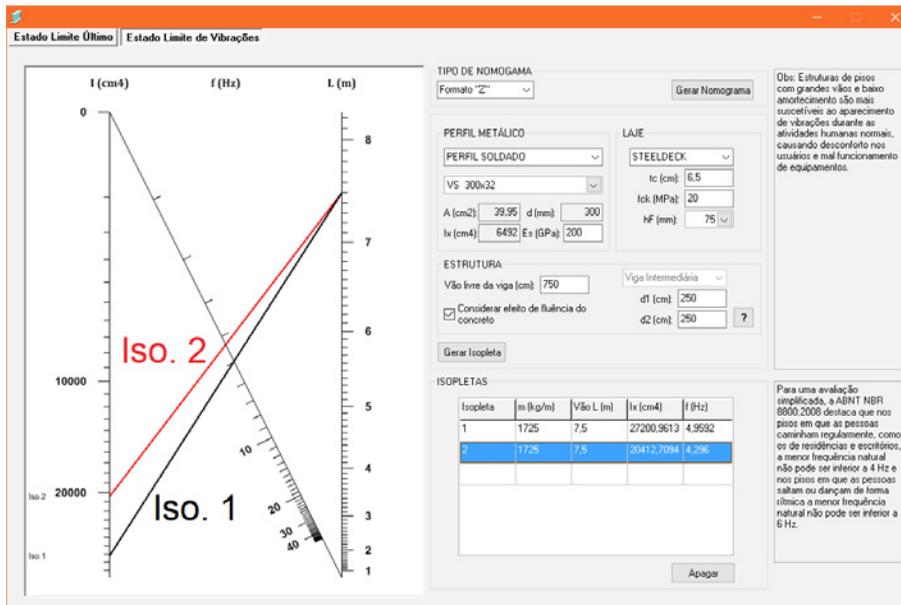
Figure 6 – Example of parallel-scale nomogram generated by the software



Source: Elaborated by the authors

After the first isopleth is drawn, the user can change any of the input data and a new isopleth will be drawn. This new situation is created by clicking on “Gerar Isopleta” (Generate Isopeth). All the isopleths can be viewed at the same time in the nomogram, as shown in Fig. 7. When tracing a new isopleth, the values of the moment of inertia, free span and fundamental frequency for each of the situations can also be visualized in the isopleth table. The distributed mass, though, is fixed for all situations.

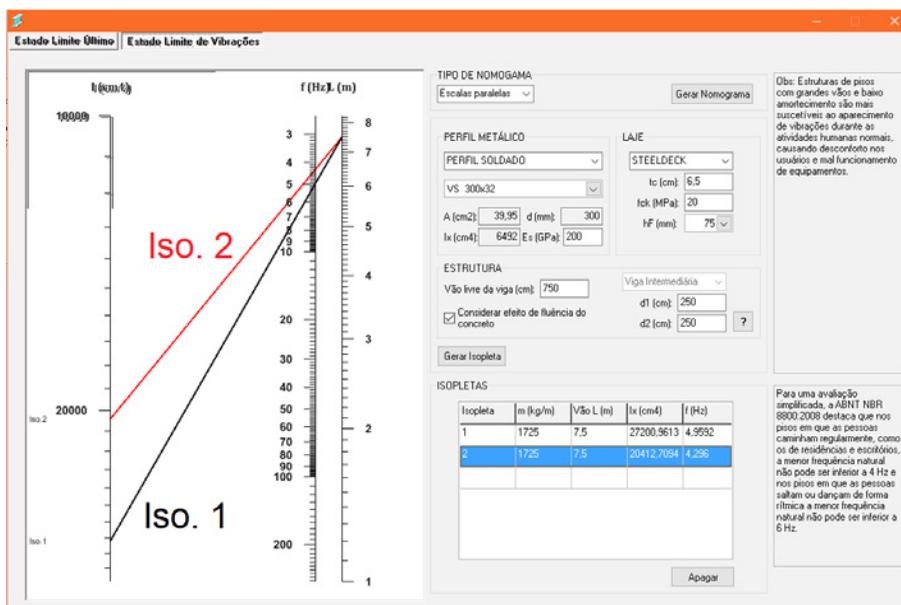
Figure 7 – New isopleth drawn in comparison to the original one



Source: Elaborated by the authors

If the new situation introduces values not covered yet in the scales of the nomogram, then a new nomogram is generated, covering all the situations contained in the isopleth table. The type of nomogram can also be modified. There are two options available: parallel-scale nomogram or “Z Chart” nomogram, Figure 8.

Figure 8 – Example of “Z Chart” nomogram generated by the software



Source: Elaborated by the authors

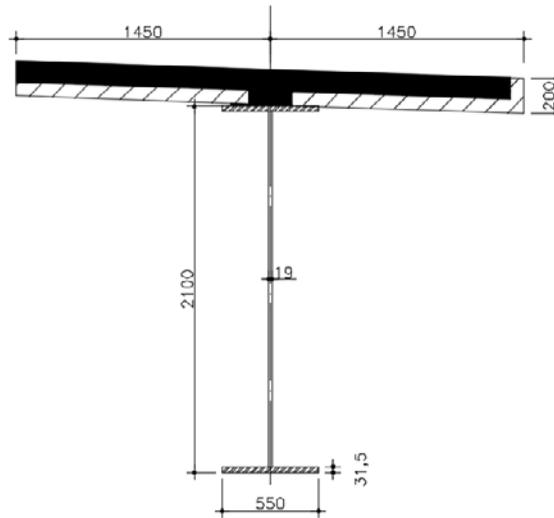
6 Case study

The present case deals with a viaduct composed of mixed beams with a span of 35 meters. It was considered an intermediate beam with welded profile type “I” of height equal to 2100 mm and width of the table, both inferior and superior, equal to 550 mm, according to Fig. 9.

There is a distance of 2900 mm between beams, as can be seen in Figure 10. A solid concrete slab was used with 200 mm in thickness and f_{ck} equal to 40 MPa. The steel presents an elastic modulus equal to 200 GPa.

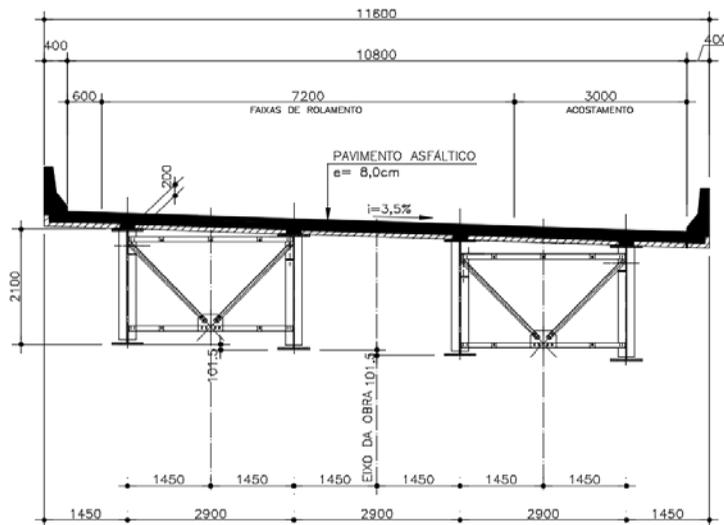
For the calculation of the fundamental frequency, the inertia of the homogenized section, calculated from the effective width of the slab equal to 2900 mm, was considered. The loads considered were only the weight of the metal profile, which has a linear mass of 573.76 kg / m, and of the slab, in a width of 2900 mm, with a concrete weight of 25 kN / m³.

Figure 9 – Cross section of the composite beam



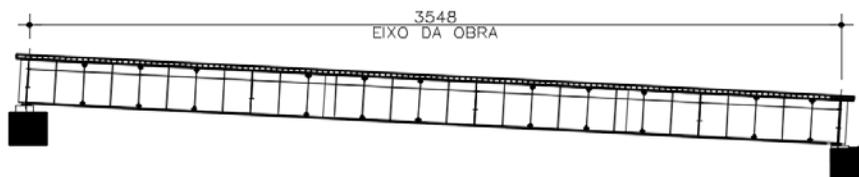
Source: Elaborated by the authors

Figure 10 – Cross section of the beam



Source: Elaborated by the authors

Figure 11 – Longitudinal section of the beam



Source: Elaborated by the authors

6.1 Analytical calculation

First, the moment of inertia of the isolated steel profile was calculated, $I_{(x,steel)}$, according to the parallel axis theorem, and the result was equal to 5044978.91 cm^4 .

For the calculation of the moment of inertia of the homogenized section, the concrete section was transformed into an equivalent section of steel by dividing its area by the homogeneity coefficient α_E . To determine this coefficient was calculated the secant elastic modulus of the concrete, E_c :

$$E_c = 0.85 \times 5600 \times \sqrt{f_{ck}} \quad (36)$$

$$E_c = 0.85 \times 5600 \times \sqrt{40} = 30104.88 \text{ MPa} \quad (37)$$

$$\alpha_E = \frac{E_s}{E_c} = \frac{200 \times 1000}{30104.88} = 6.64 \quad (38)$$

With an effective width of 290 cm and 20 cm in thickness, the concrete area of the slab is 5800 cm^2 , so, considering the transformed section, the area is equal to 873.49 cm^2 . The theorem of the parallel axes was applied to calculate the moment of inertia I_x of the transformed section and the result was equal to 10347003.74 cm^4 .

The distributed mass considered for the calculation of the fundamental frequency was the weight of the metal profile added to the mass of the concrete along the slab effective width, being equal to 2023.76 kg/m .

Applying Eq. (24), with k_n equal to π^2 , it is found that:

$$f = \frac{\pi}{2} \sqrt{\frac{2000 \times 10347003.74}{2023.76 \times 35^4}} = 4.10 \text{ Hz} \quad (39)$$

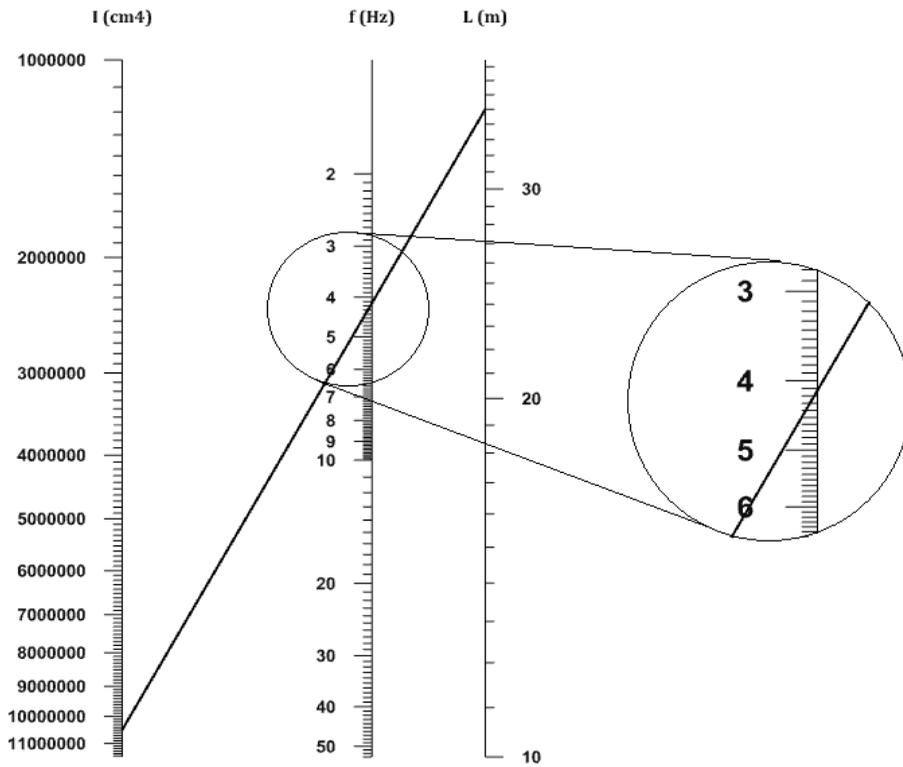
6.2 Results and discussion

The results of the analytical calculation for the fundamental frequency of the beam under study were 4.10 Hz. The nomograms, type parallel-scale and type “Z”, generated by the IVM, is presented in Figs.12 and 13. When analyzing the nomograms generated by the program, the same result is found in the analytical calculation, but it is not possible to evaluate the precision in the decimal places.

The inaccuracy is on the order of 10^{-2} Hz in the parallel-scale nomograms and 10^{-1} Hz in the “Z Chart” nomograms. depends on the adopted scale, limit of vibrations, since it is possible to identify the limits established by norm and verify the conformity of the results found with these limits. Therefore, nomography can be an efficient tool for calculating the fundamental frequency of steel-concrete composite beams. In addition, the same nomogram allows evaluating the value of the frequency under different configurations of beams, which confers great convenience in the use of this tool.

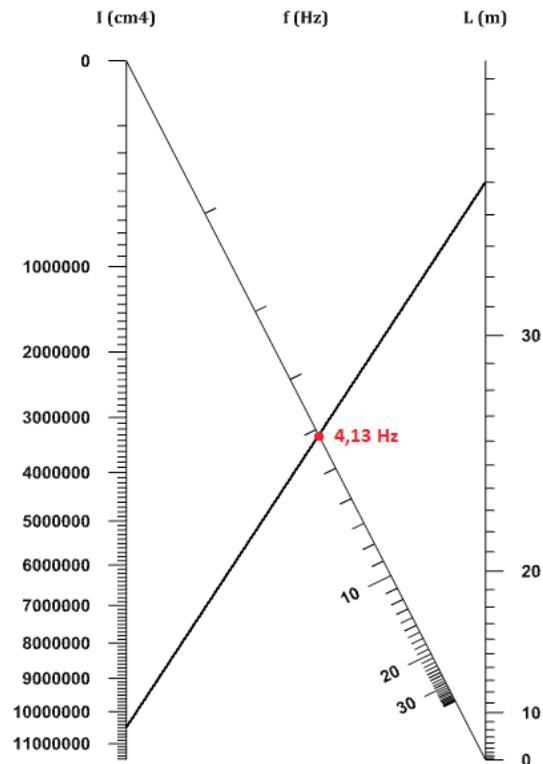
It is important to notice that the program does not take into account the effect that transverse stiffeners or other elements may cause on the rigidity or damping capacity of the structure. They may result in different frequency values so that a different approach would be required in this case.

Figure 12 – Parallel-scale nomogram generated by the program



Source: Elaborated by the authors

Figure 13 – “Z chart” nomogram generated by the program



Source: Elaborated by the authors

7 Conclusion

Structures with composite systems require the verification of the limit state of vibrations, as pointed out previously in this work. This verification is based on dynamic analysis of structures, at which the fundamental frequency is one of the main parameters to be checked.

Therefore, this work intended to develop a computer program that would enable users to calculate the fundamental frequency of steel-concrete composite beams by using nomograms. Besides, it also aimed to show the efficiency and practicality of this tool.

The results were satisfactory since it was developed an application with an intuitive interface, at which the user can generate a nomogram for calculating the fundamental frequency of composite beams. The nomogram enables the user to easily analyze the effects that a change in the input data may cause on the frequency values of the structure. This process allows the user to select the most suitable parameters for the beam, so that, vibration problems may be avoided efficiently.

Two types of nomograms were implemented: parallel-scale nomogram and “Z chart” nomogram. Both types proved to be efficient in the results, as shown in the case study. The adopted scales were accurate, so that, this tool can be used to check the limit state of vibration of composite beams.

The application was integrated into a program for designing composite beams, becoming a more complete platform, which was called “IVM”. The program may also be useful for researchers as they develop new features.

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References

- ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. **ABNT NBR 8800**: projeto de estruturas de aço e de estruturas mistas de aço e concreto de edifícios. Rio de Janeiro: ABNT, 2008.
- CLOUGH, R. W. & PENZIEN, J. **Dynamics of structures**. 3. ed. Berkeley: Computers & Structures, 2003.
- DOERFLER, R. On Jargon: the lost art of nomography. **The UMAP Journal**, Bedford, v. 30, iss. 4, p. 457-493, apr. 2009.
- ELLINGWOOD, B.; TAILLIN, A. Structural serviceability: floor vibrations. **Journal of Structural Engineering**, Reston, v.1 10, iss. 2, p. 401-418, feb. 1984.
- FENGHAO SUN, Ke Ma et al. A nomogram to predict prognosis after surgery in early stage non-small cell lung cancer in elderly patients. **International Journal of Surgery**, London, v. 42, p. 11-16, 2017.
- FLAHERMAN, Valerie J. et al. Early weight loss nomograms for exclusively breastfed newborns. **Pediatrics**, Virginia, v. 135, iss 1, 2014. Available at: <DOI:10.1542/peds.2014-1532>. Accessed: 17 dez. 2017
- HICKS, S. J.; SMITH, A. L.; DEVINE, P. J. **Design of floors for vibration**: a new approach. Berkshire: The Steel Construction Institute, 2009.
- KARLO, Christoph A. et al. Renal cell carcinoma: a nomogram for the CT imaging-inclusive prediction of indolent, non-clear cell renal cortical tumours. **European Journal of Cancer**, Amsterdã, v. 59, p. 57-64. 2016.
- NÓBREGA, P. G. B. **Análise dinâmica de estruturas de concreto**: estudo experimental e numérico das condições de contorno de estruturas pré-moldadas. 2004. 284 f. Tese (Doutorado em Engenharia de Estruturas) - Escola de Engenharia de São Carlos, São Carlos, 2004.
- PLEIJHUIS, R. G. A validated web-based nomogram for predicting positive surgical margins following breast-conserving surgery as a preoperative tool for clinical decision-making. **The Breast**, Amsterdã, v. 22, issue 5, p. 773-779. 2013.
- SILVA JÚNIOR, L. A. **Análise experimental de vibrações em sistemas de piso misto de aço e concreto e avaliação de desempenho considerado o conforto humano**. 2015. 190 f. Dissertação (Mestrado em Engenharia Civil) - Universidade Federal de Viçosa, Viçosa, 2015.

Paulo Gomes da Costa Filho, Calebe Paiva Gomes de Souza, Wesley Calland Serra de Almeida

SILVA, S. S. **Vibration characteristics of steel-deck composite floor systems under human excitation**. 2007. Tese (Ph.D.) - Queensland University of Technology, Brisbane, 2007.

SOMMER, R. M. R. **Análise de vibrações em pisos mistos aço/concreto**. 2002. 149 f. Dissertação (Mestrado em Engenharia de Estruturas) - Universidade Federal de Minas Gerais, Belo Horizonte.

ZHANG, Zhongheng; KATTAN, Michael W. Drawing nomograms with R: applications to categorical outcome and survival data. **Ann. Transl. Med.**, Maryland, v. 5, iss. 10, p.211, 2017. Available at: DOI:10.21037/atm.2017.04.01. Accessed: 13 nov. 2017.

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