The use of Green's functions for the transient analysis of a thermoresistive pyranometer

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Resumo

Neste artigo o comportamento térmico de um piranômetro sob condições transientes é analisado utilizando-se da técnica das funções de Green. A solução transiente do campo de temperatura foi obtida como função de dados experimentais levantados com o instrumento operando em condições reais. Os resultados mostraram a adequacidade do método das funções de Green quando se requer soluções de problemas de condução transiente associados às condições de contorno que simultaneamente evoluem com o tempo.

Palavras-chave: *Piranômetro. Funções de Green.*

Abstract

In this paper the thermal behavior of a pyranometer under transient operation is analysed through the use of the technique of Green's functions. The transient solution for the field of temperature was obtained as a function of experimental data took from the instrument operating under real conditions. The results showed the feasibility of the method of Green's functions when it is necessary to have solution of transient conduction problems associated to boundary conditions that evolves with time.

Keywords: *Pyranometer. Green's functions.*

1 Introduction

Measurement of heat flux is essential for the characterization and control of many thermal processes. All surface mounted heat flux transducers in some way distort the temperature field and heat flux path which they are designed to measure. As product of studies for more than five decades, there are many types of sensors with common use in heat flux measurements (GORDON, 1953 and VAN HEINIGEN et al., 1985). This kind of sensor is widely used in industrial and scientific applications. However in the literature there are few publication which works with the adequate mathematical formulation of the heat transfer process occurring during its use or aiming to determine its thermal performance. The major difficulty is to obtain the theoretical solution that eventually will permit the analysis of the sensor's thermal behavior under real time operation.

The present study was stimulated by the necessity of designing robust heat flux sensors and to better understand the effects of the sensor's body thickness in their transient and frequency responses. ASH (1969) proposed a method in which the response characteristics of a thin foil heat flux sensor were obtained from the knowledge of the temperature distribution on the sensor's surface. PRASAD and MOHANTY (1979) made an analysis of a foil heat flux sensor in which they included a theoretical estimation of the sensitivity and they detailed the region where the electromotive force produced by the heat flux sensor is linear to the measured value of the convective heat transfer on the Gardon's type heat flux transducer. The utilization of this kind of analysis permit the establishment of the reference temperature in experiments of convective heat transfer. BORODIN et al. (1983) investigated the sensitivity and response time of thermoelectric and thermomagnetic film sensors. By the temperature distribution on sensor's body, they concluded that the sensitivity of circular thermoelectric sensors do not change for disc radius greater than 3 mm.

BECK and WEDEKIND (1986) presented results of a research in which they evaluated the effective surface of an absorption layer subjected to a non-uniform radiant heat flux. Their analysis suggests that if a transducer is projected in an appropriate way, with a convenient experimental technique, the history temperature-time of the sensor will eventually conduct to the determination of the absorbed energy by the sensor no matter if the sensor has been placed in an arbitrary position.

As highlighted by FENG and MICHAELIDES (1997) the method of Green's function has been advocated for obtaining the entire temperature field and average physical properties during transient heat conduction processes. Using Green's functions in heat transfer problems has several advantages because it is a powerful and flexible method, since the derived Green's functions for a given geometry may be used in conjunction with a variety of initial and boundary conditions. Still after Feng and Michaelides, the systematic procedure is available for obtaining the functions and the one dimensional obtained Green's functions may be used as building blocks to obtain two and three dimensional solutions to unusually more complex problems. COLE and YEN (2001) studied Green's functions for a rectangle as an apllicative example of heat transfer processes in terms of exponentials which have better numerical properties than hyperbolic functions.

The problem of transient heat conduction was recently studied by JANICKI et al. (2002) and BECK and MCMASTERS (2004)**.** Janicki et al. presented an approach to the method of transient thermal states in electronic circuits using an analytical solution of the heat equation. Fully three-dimensional analytical time dependent solutions were determined with the help of Green's functions. The solution method was illustrated in detail on a practical example, where the results of transient thermal simulations of a real hybrid circuit was compared with infrared measurements. On the other hand, Beck and McMasters proposed an analytical solution for the problem of transient thermal conduction with body movement for an orthotropic parallelepiped. The solution uses two types of Green's functions: one coming from the Laplace transform method and the other from the method of separation of variables. An example was given for a multi-dimensional case involving both prescribed heat flux and temperature boundary conditions. GRINE et al. (2006) used the Green's functions method to determine the analytical solutions of the transient conduction in a flat plate subjected to a variable heat flux. WANG and TAN (2006) proposed the solution for transient heat conduction in concrete filled steel circular hollow sections subjected to fire, using Green's functions. The method worked as well as the more sophisticated numerical codes presently used.

The main objective of the present study is to understand the transient behavior of solar radiometers in general and it is specifically justified by the necessity to improve the project of a thermoresistive pyranometer early developed by DE LIMA (1987). The established governing equation for the sensor of such instrument was coupled with the transient boundary conditions took under the pyranometer's real operation. An analytical solution was presented for temperature distribution across the sensor's body and the method of Green's functions for this kind of problem was sucessfully used.

2 Statement of the problem

Figure 1 shows the sensing body of the studied pyranometer which is an instrument used for the measurement of global solar radiation. It consists of a glass hemisphere with 3.5 cm inner radius and 0.5 mm thick. Inside the glass hemisphere there is a platinum thermoresistive solar sensor fused and black painted on a pyrex substract whose surface area is 2.8 x 2.8 square centimeter and 1 mm thick.

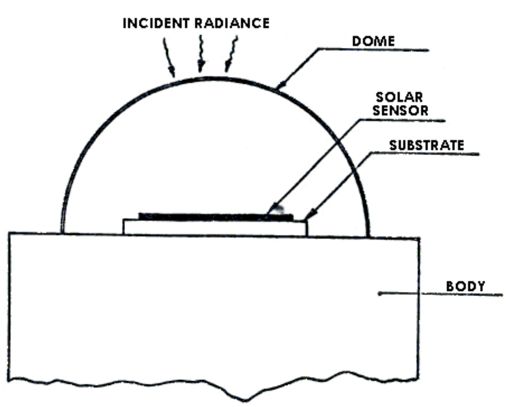

Figure 1: Sensing body of the pyranometer.

Adopting the incident radiation heat flux constant and uniform over the solar sensor's surface and neglecting lateral heat losses on the sensor's body, its transient temperature response can be expressed by the following heat transfer problem:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

where T is the temperature (K), t is the time (s), α is the thermal diffusivity (m²/s) and x is the normal length to surface (m). As stressed by JANICKI et al. (2002) there are three basic kind of boundary conditions describing the heat exchange at the structure boundaries: Dirichlet condition (prescribed temperature), Neumann condition (prescribed heat flux) or Robin condition (convective condition). The radiation heat flux on the surface of the pyranometer is conducted towards the structure of the solar sensor and removed by means of convection and re-radiation which in the present case is neglected. Thus, based on Fourier's and Newton's law, the heat removal at the sensor surface can be represented optimally by the third kind of boundary conditions and corresponding initial condition.

In the region R ($0 \le x \le L$), the equation for the temperature distribution along the sensor's thickness and associated boundary conditions are as follow:

$$-\frac{\partial T}{\partial x} + HT = \frac{q''}{K} \quad x = 0\ ,\ t > 0 \tag{1.a}$$

$$T = f(t) \quad x = L\ ,\ t > 0 \tag{1.b}$$

and the initial condition

$$T(x,0) = T_0 \quad \text{in } R \tag{1.c}$$

where K is the thermal conductivity (W/m.°C), q" is the incident heat flux (W/m²) and H = h / K, being h the convective heat transfer coefficient (W/m².°C). The function f (t) is the boundary condition obtained experimentally through the application of a radiant heat flux of 1000 W/m² at the initial ambient temperature of 23.5°C.

Figure 2 presents the transient temperature distribution on the upper surface of the sensor (x = L) after a radiation pulse. The adjusted curve was obtained through the fitting of the measured experimental data. This distribution represents the boundary condition (1.b).

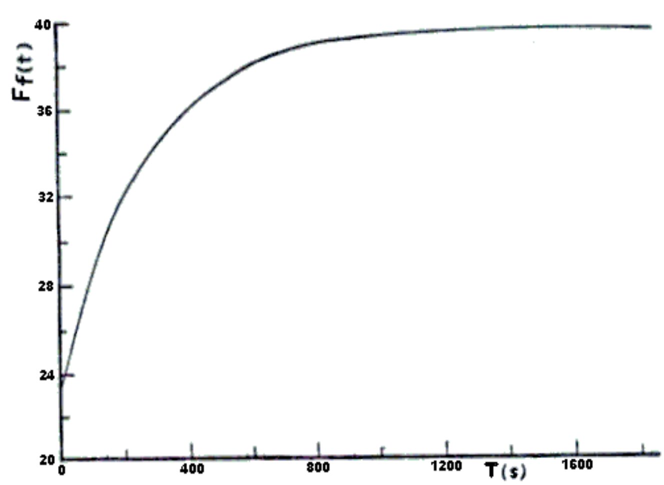

Figure 2: Temperature rise over the ambient temperature versus time on the sensor's upper surface

The fitted expression for the boundary condition f (t) is

$$f(t) = 23 + 16.66\,(1 - e^{-t/255}) \tag{2}$$

The resolution of the differential equation (1) in terms of the Green's function G (x,t / x',τ) requires the solution of the following homogeneous associated problem:

$$\frac{\partial^2 G}{\partial x^2}(x,t/x',\tau) + \frac{1}{\alpha}\delta\,(x-x')\,\delta(t-\tau) = \frac{1}{\alpha}\frac{\partial \mathrm{G}}{\partial \mathrm{t}}(x,t/x',\tau) \tag{3}$$

submitted to the boundary conditions which represent the homogeneous version of the boundary conditions from the original problem, i.e.,

$$-\frac{\partial G}{\partial x}(x,t/x',\tau) + HG(x,t/x',\tau) = 0 \qquad \text{in } x = 0,\ t > \tau \tag{3.a}$$

$$G(x,t/x',\tau) = 0 \qquad \text{in } x = L,\ t > \tau \tag{3.b}$$

where δ is the Dirac delta function.

Once the solution of the homogeneous associated problem is known, the solution of problem (1) can be written as

$$T(x,t) = \int_0^L G\ .T_0 dx' + \alpha\int_{\tau=0}^{t} \left.\frac{\partial \mathrm{G}}{\partial \mathrm{x}}\right|_{x=L} f(t)d\tau + \alpha\int_0^t G\Big|_{x'=0} \frac{q''}{k\,.\,d\tau} \tag{4}$$

The Green's function defined by equation (3) is determined through the solution of the homogeneous version of the problem (1), for the same region R and given by

$$\frac{\partial \psi^2}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \psi}{\partial \mathrm{t}} \qquad \text{in R} \tag{5}$$

submitted to the boundary conditions:

$$\psi = 0 \qquad \text{in } x = L,\ t \tag{5.a}$$

$$\frac{\partial \psi}{\partial x} = H\psi \qquad \text{in } x = 0,\ t \tag{5.b}$$

and to the initial condition:

$$\psi = T_0 \qquad \text{in R for } t = 0 \tag{5.c}$$

Using the technique of variables separation, the solution of equation (5) is obtained as

$$\psi(x,t) = \sum_{n=1}^{\infty} \mathrm{e}^{-\alpha\lambda_n{}^2 t} D_n \sin[\lambda_\mathrm{n}\,(L-x)] \quad \int_0^L T_0 \sin[\lambda_n\,(1-x)]d\,\mathrm{x}' \tag{6}$$

where λ_n are the associated eigenvalues obtained from the transcendental equation

$$L\,.\lambda_n\,\mathrm{cotg}(\lambda_n L) = -H\,.L \tag{7}$$

and

$$D_n = \frac{2(\lambda_n{}^2 + H^2)}{L\,.(\lambda_n{}^2 + H^2) + H}$$

h(7.a)

Comparing the equation (6) to the general solution

$$\psi(x,t) = \int_0^L G(x,t/x',\tau)\Big|_{\tau=0} T_0 \, d\, x' \tag{8}$$

From auxiliary problem (5), one obtains

$$G(x,t/x',\tau)\Big|_{\tau=0} = \sum_{n=1}^{\infty} D_n \, e^{-\alpha\lambda_n^2 t} \, sin[\lambda_n(L-x)] \; sin[\lambda_n(L-x')] \tag{9}$$

Substituting on equation (4) the values of $\frac{\partial G}{\partial x'}\Big|_{x'=L}$ e $G\Big|_{x'=0}$ found in the equation (9), the expression for the field of temperature along the thickness L of the sensor's substract is obtained as follow:

$$T(x,t) = T_0 \sum_{n=1}^{\infty} D_n \, e^{-\alpha\lambda_n^2 t} \frac{sin[\lambda_n(L-x)][1-\cos(\lambda_n L)}{\lambda_n} + \sum_{n=1}^{\infty} D_n \frac{sin[\lambda_n(L-x)] f\ (t)}{\lambda_n} + \sum_{n=1}^{\infty} \tag{10}$$

$$D_n \frac{sin[\lambda_n(L-x] \; \sin[\lambda_n L]}{\lambda_n^2} \frac{q''}{k} + \sum_{n=1}^{\infty} D_n \frac{sin[\lambda_n(L-x)] \, e^{-\alpha\lambda_n^2 t}}{\lambda_n} \{-f(0) - \int_0^t e^{\alpha\lambda_n^2\tau} \, . f'(\tau) d\tau -$$

$$-\frac{q''}{k} \frac{\sin[\lambda_n L]}{\lambda_n}\}$$

Applying the boundary condition in x = L, it is observed that the right hand side of the equation (10) will be zero, not satisfying equation (1.a). This difficulty can be avoided through the establishment of the closed expressions for this kind of series (as seen in the Appendix A) or simply by observing the convergence of solution on the boundary proximity, as shown in Figure 3.

3 Results and discussion

Figure 3 presents the transient response of the sensor's substract on different layers. The thickness of the substract was of 10 mm. Taking as for example the layer where x = 0.9 the layer accuses the temperature of about 32º C when the ambient temperature was of 23.5°C and the radiation pulse of 1000 W/m² .

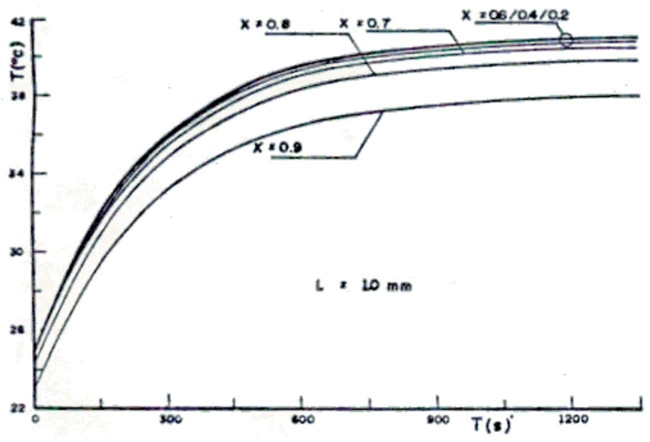

Figure 3: Transient response of the sensor's substract on different layers.

Figure 4 presents the pyranometer's response for different values of the substract's thickness. It is observed that for values of thickness greater than 1mm, the pyranometer's time response do not change, being about 250s for 63% of the steady-state response. This value is considered to high for transient investigation, suggesting structural modification on the substract's material, as for the improvement of the instrument.

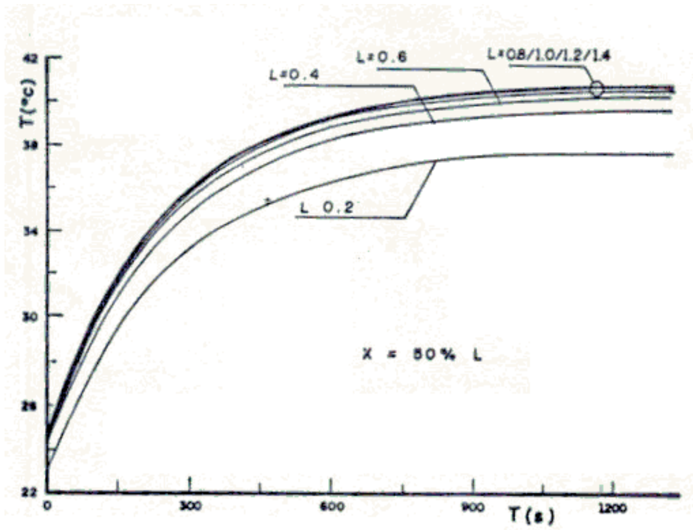

Figure 4: Transient response of the sensor for different substract's thickness

In the hypothesis of using substract with greater diffusivity, as for example metals it is observed that the thickness of the sensor will not be a stringent factor.

4 Conclusion

It is shown the method adjustment of Green function when it is required problem solutions in transient conduction associated to boundary conditions that evolves with time. The use of variable separation method in transient boundary conditions in this case is impracticable.

The solution for the temperature distribution presents the transient response for pyranometer, allowing yet a estimate of response time. From the temperature distribution, a change in substract, with a consequent increase in thermic diffusivity, will decrease the response time without excess decrease of its thickness. A relationship between transversal area and thickness in substract can be obtained using a analog procedure, but a bidimensional analysis in substract is necessary.

References

ASH, R. L. Response characteristics of thin foil heat flux sensors. *AIAA Journal*, Reston, v. 7, p. 2332-2335, 1969.

BECK, J. V.; MCMASTERS, R. L. Solutions for multi-dimensional transient heat conduction with solid body motion. *International Journal of Heat and Mass Transfer*, London, v. 47, p. 3757-3768, 2004.

BECK, J. V.; WEDEKIND, G. L. Characteristics of a simple energy absorption transducer. *Journal of Heat Transfer*, New York, v. 108, p. 676-683, 1986.

BORODIN, V. V.; GELFGAT, D. M.; DASHEVSKIL, Z. M. Film-type thermoelectric and thermomagnetic thermal radiation sensor and their output parameters. *Inzhenerno-Fizicheskii Zhurnal*, Minsk, v. 44, p. 573-580, 1983.

COLE, K. D.; YEN, D. H. Y. Green's functions, temperature and heat flux in the rectangle. *International Journal of Heat and Mass Transfer*, London, v. 44, p. 3883-3894, 2001.

DE LIMA, L. C. Um piranômetro com sensores de platina. *Ciência e Cultura*, Campinas, SP, v. 39, p. 39-43, 1987.

FENG, Z. G.; MICHAELIDES, E. E. The use of modified Green's functions in unsteady heat transfer. International Journal of Heat and Mass Transfer, London, v. 40, p. 2997-3002, 1997.

GORDON, R. An instrument for the direct measurement of intense thermal radiation. *Review of Scientific Instrument*, Argonne, v. 24, p. 366-370, 1953.

GRINE, A.; DESMONS, J.Y.; HARMAND, S. Models for transient conduction in a flat plate s ubjected to a variable heat flux. *Applied Thermal Engineering*, London, Sep. 2006. In press.

JANICKI, M.; DE MEY, G.; NAPIERALSKI, A. Application of Green's function for analysis of transient thermal states in electronic circuits. Microelectronics Journal, London, v. 33, p. 733-738, 2002.

PRASAD, B. V. S.; MOHANTY, A. K. Analysis and calibration of a foil heat flux sensor for convective heat loss from a solid surface to an airstream. *Journal of Physics and Scientific Instrument*, Bristol, v. 12, p. 270-271, 1979.

VAN HEINIGEN, A. R. P.; DOUGLAS, W. J. M.; MUJUNDIN, A. S. A high sensitivity, fast response heat flux sensor. *International Journal of Heat and Mass Transfer*, London, v. 28, p. 1657-1667, 1985.

WANG, Z. H.; TAN, K. H. Green's function solution for transient heat conduction in concrete-filled CHS subjected to fire. *Engineering Structures*, London, v. 28, n. 11, p. 1574-1585, 2006.

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Appendix A. Derivation of a closed form for the series

$$\sum_{n=1}^{\infty} \frac{D_n \,.\sin\lambda_n (L-x)\,(1-\cos\lambda_n L)}{\lambda_n}$$

Considering the follow problem of heat conduction

$$\frac{\partial^2\phi}{\partial x^2} = \frac{1}{\alpha}\,\frac{\partial\phi}{\partial t} \qquad 0 < x < L,\ t > 0 \qquad \text{(A.1.a)}$$

$$\phi = 0 \qquad x = L,\ t > 0 \qquad \text{(A.1.b)}$$

$$\frac{\partial\phi}{\partial x} = H\phi \qquad x = 0,\ t > 0 \qquad \text{(A.1.c)}$$

$$\phi = 1 \qquad 0 < x < L,\ t = 0 \qquad \text{(A.1.d)}$$

the solution is given by

$$\phi(x,t) = \sum_{n=1}^{\infty} e^{-\alpha\lambda_n^2 t}\,.D_n\,.\sin[\lambda_n(L-x)]\,.\int_0^L 1.\sin[\lambda_n(L-x')\,]dx' \qquad \text{(A.2.a)}$$

where

$$L\lambda_n \; cotg\,\lambda_n L = -H \qquad \text{(A.2.b)}$$

and

$$D_n = \frac{2(\lambda_n^2 + H^2)}{L(\lambda_n^{\,2} + H^2) + H} \qquad \text{(A.2.c)}$$

The equation (2) must satisfy the initial condition A.1.d which results

$$\sum_{n=1}^{\infty} \frac{D_n \,.\sin\lambda_n (L-x)(1-\cos\lambda_n L)}{\lambda_n} = 1$$

By this analogs proceedings, it can obtained the closed forms from different series that Are related to the equation (10).