Measurement and uncertainty determination during inspection of free form profiles

Medição e incerteza na determinação durante a inspeção de perfis de forma livre

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Resumo

Os avanços da tecnologia de fabricação têm possibilitado o uso crescente de superfícies de forma livre em produtos como pás de turbinas hidráulicas e lentes especiais denominadas *asféricas*. Essas superfícies apresentam algumas complicações quanto ao ajuste dos dados aos modelos matemáticos que descrevem sua geometria. Alguns métodos sugeridos pela literatura envolvem a aplicação de B-splines e B-splines não uniformes racionais (NURBS), estas bastante usadas e recomendadas para a modelagem matemática das superfícies de forma livre. O presente trabalho investiga o problema do ajuste dos dados às curvas com forma livre e apresenta uma discussão sobre a determinação da incerteza relacionada. A investigação da incerteza relacionada às curvas NURBS foi feita seguindo as recomendações do Guia ISO para expressão da incerteza de medição, ISO GUM. Uma vez que a representação matemática de curvas parametrizadas, como as NURBS, é complexa, a determinação das derivadas parciais do modelo não é adequada e uma abordagem recente sugerida pela ISO GUM, envolvendo a simulação de Monte Carlo, foi aplicada. Alguns parâmetros das curvas NURBS foram deliberadamente modificados durante a simulação de Monte Carlo, tal como o número e a localização dos pontos de controle e os valores dos nós. O método foi implementado e as curvas NURBS dos perfis estudados foram estimadas e associadas à incerteza de sua localização.

Palavras-chave: Superfície de Forma Livre. Incerteza, Inspeção. Simulação de Monte Carlo.

Abstract

Advances in manufacturing technology are allowing the growing use of *f*ree form surfaces in products like hydraulic turbines runners and aspheric optical lenses. These free form surfaces present some complications when fitting data to a mathematical model describing their geometry. Some methods have been suggested in literature applying B-splines and Non-Uniform Rational B-Splines (NURBS) mathematical models are the most suitable to fit data. This work deals with the problem of fitting data to free form curves and presents a discussion about the uncertainty determination. An investigation of the uncertainty of NURBS curves was carried out following the recommendations of the ISO Guide to the Expression of Uncertainty in Measurement, ISO GUM. As the mathematical representation of parametric curves like NURBS are somewhat complex, the determination of partial derivatives as argued by ISO GUM is unsuitable. The new ISO GUM approach of Monte Carlo simulation is then used. Some parameters of the NURBS curves were deliberated changed during simulation, such as the number and the localization of the control points and the values of the knots. The implementation was carried out and the uncertainties of the profile curves were estimated.

Keywords: Free form surfaces. Uncertainty. Inspection. Monte Carlo simulation.

1 Introduction

A free form surface may be considered as a kind of surface that cannot be fitted by classical analytical shapes, like planes, circles and spheres, and it is generally defined by a set of control points. Advances in manufacturing technology are allowing the specification of free form surfaces in several products ranging from hydraulic turbines runners, aspheric optical lenses, molds and dies for aerospace and automotive industries.

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The quality of the parts having free form surfaces is verified by comparing designed and manufactured surfaces and thus determining the errors of the part produced. This verification is carried out by performing the following steps: measurement of the part surface to acquire data from its localization and dimensioning, a description of the surface by a mathematical method and adopting an approach to compare data from fitted surface with the theoretical designed surface (Li and Gu, 2004).

The measurement step was initially performed by using mechanical gauges and templates but it had demanded high skilled technicians to its implementation. The advent of Coordinate Measuring Machines (CMM) allowed the improvement of measurement performance as accuracy was increased and time spent in inspection was reduced. Measurement with CMMs is nowadays carried out by using two different techniques to capture data points on a given part surface, contact and non-contact Techniques. Contact techniques are associated to determination of Cartesian coordinates of points by physically touching the surface of the part with the CMM stylus. Non-contact methods are related to the use of optical or laser probes on CMMs to acquire a large amount of data points (point cloud) without physically touching the surface (Li and Gu, 2004; Savio *et al*., 2007).

The description of the surface is carried out with data acquired in measurement and it has fundamental importance because all subsequent steps in inspection depend on these results. It is performed by fitting measured data points to curves or surfaces by using computational algorithms. Depending on the amount of data, a different approach may be used to fit a mathematical model that describes the geometry of the surface.

The comparison of two free form surfaces is accomplished by building a reference position and putting both surfaces at this location at the same orientation. The first step is the localization or registration of the surfaces, where the data points are fitted to curves and surfaces and its orientation and localization is well defined. The second step is the comparison, where distances between models designed and fitted are computed by iterative processes.

Localization was initially established by using polyhedral approximations and representations with polygons. Later, higher order forms were adopted to represent the surfaces, as parametric description using Splines, Coons, Bézier, B-spline and NURBS (Non-Uniform Rational B-Splines) surface models. These models are most used to fit data obtained from contact measurement but it is also used to adjust surfaces from cloud of points after a pre-processing of data from non-contact measurement. The pre-processing involves eliminating outliers and noise before fitting NURBS curves and surfaces. The last step is to perform the rotation and the translation of the model to adjust orientation in relation to the designed model (reference) (Li and Gu, 2004; Savio *et al*., 2007).

Comparison involves the determination of the distance between the points in the designed model and the respective points in the fitted model. When fitting NURBS models, it is required to find a point in the NURBS parametric surface such as the determined distance has a minimum value. This is usually carried out by using computational iterative methods. The distance may also be determined by calculating distance between a point in the designed model and a plane in the fitted one (Li and Gu, 2004).

There are recent developments in literature regarding the measurement and inspection of free form surfaces. Ristic *et al*. (2004) presented a case study of a turbine runner recovering, that the free form surfaces of the runner were measured separately and NURBS models were used to fit the surfaces. Bagci (2009) presented three case studies involving the reverse engineering of parts having free form surfaces. The author applied techniques of Computer-Aided Design (CAD) and Computer-Aided Manufacturing (CAM) to recover the design of broken parts.

Some works in literature dealt with application of the ISO Guide to the Expression of Uncertainty in Measurement (ISO GUM, 2008) to determine uncertainty of profiles with straight geometry (Ribeiro *et al*., 2003). Bachmann et al. (2003) developed the concept of statistical confidence bounds (SCB) to determine the uncertainty of a given surface or profile. The authors recommended this approach to solve geometric problems like straight lines and planes, having linear models, with standard errors of the parameters determined by eigenanalysis of the covariance matrix. Danish et al. (2006) developed a study of the effect of the CMM point coordinate uncertainty when fitting circular features. The Monte Carlo simulation was used to determine the uncertainty of circular features. Kruth et al. (2009) proposed a similar approach to determine the uncertainty of CMM including estimates of the circular form deviations. Barini et al. (2010) presented an analysis of uncertainty of a complex surface measured with a CMM with a contact probe. This approach was implemented using design of experiments and its implementation was in agreement with the ISO However, it was observed that new developed approaches rely on the assistance in the task of uncertainty determination.

The present work deals with the problem of fitting data to free form curves and presents a discussion about the

problem of uncertainty determination. An investigation of the uncertainty of NURBS curves was carried out following the recommendations of the ISO Guide to the Expression of Uncertainty in Measurement, ISO GUM (2008). As the mathematical representation of parametric surfaces with NURBS models are complex, the determination of partial derivatives like argued by ISO GUM is unsuitable. The ISO GUM approach of Monte Carlo simulation is then used (ISO GUM Supplement, 2008). Some parameters of the NURBS curves were changed during simulation, as the number and the localization of the control points and knots vector. The implementation was carried out and the uncertainties of the curves were estimated.

2 NURBS freeform curves

Most of the CAD Software nowadays is using Non-Uniform Rational B-Splines curves (NURBS) to fit data to the free form curves and surfaces. This tool in CAD design is a generalization of B-Splines and rational Bézier curves and surfaces and its promising use is related to the flexibility in designing a large amount of geometries as standard analytical shapes and free form shapes, assisted by numerically stable and accurate algorithms (Piegl and Tiller, 1997).

NURBS curves are considered as vector-valued piecewise rational polynomial functions. Its definition requires the specification of control points and the weights associated to each one. Equation 1 shows mathematical model of a NURBS curve, where w(i) are the weights of the control points vector P(i) and $N_{i,k}(u)$ are the normalized B-spline basis functions of degree k.

$$C(u) = \frac{\sum_{i=0}^{n}\left(w(i)\cdot P(i)\cdot N_{i,k}(u)\right)}{\sum_{i=0}^{n}\left(w(i)\cdot N_{i,k}(u)\right)} \qquad (1)$$

These B-splines basis functions are defined as represented in equation 2, where t_i are the knots of the knot vector $T=[t_0, t_1, …, t_m]$. In this expression, when k=1, $N_{i,1}(u)$ may assume the value 1 when $t_i \le u \le t_{i+1}$ or be 0 otherwise. It is important to consider that the number of knots (m+1) may be related to degree (k) and to the number of control points (n+1) by $m = n + k + 1$. It must be observed that the sequence of knots defined by U must be non-decreasing, representing the intervals to determine the segments of a curve. The knots spacing is not necessarily periodic, as admitted having different values and multiplicities each, that means the B-spline may admit different degrees at each segment. Figure 1 shows a NURBS curve with degree k=3, with 6 control points and having knots vector equals to T=[1 1 1 3 4 7 9 9 9].

$$\begin{cases} N_{i,1}(u) = \begin{cases} 1 & if \quad t_i \le t \le t_{i+1} \\ 0 & otherwise \end{cases} \\ N_{i,k}(u) = \dfrac{u - t_i}{t_{i+k} - t_{i+1}} \cdot N_{i,k-1}(u) + \dfrac{t_{i+k+1} - u}{t_{i+k+1} - t_{i+1}} \cdot N_{i+1,k-1}(u) \end{cases} \qquad (2)$$
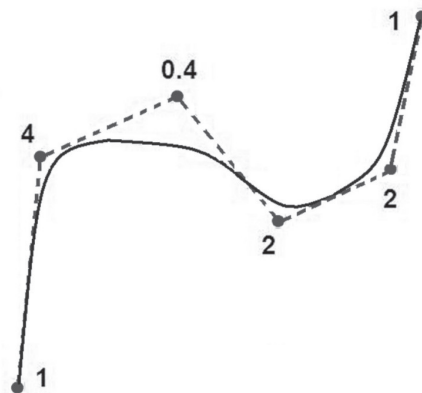

Figure 1. NURBS curve with its control points (adapted from Piegl and Tiller, 1997).

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When fitting experimental data to NURBS curves and surfaces, two different approaches may be applied: interpolation and approximation. Interpolation means that the fitted curve is passing through all data points and approximation means that a curve is fitted to a given data points but it is not passing through all the points, having an error associated to the whole process. The interpolation of data points can be performed by global or local approaches (Piegl and Tiller, 1997).

The method of approximation is used when dealing with data captured by CMM and other measuring devices, as there are a large number of points representing a given surface or profile and some error sources are expected to introduce variability in point positions (noise). It is a more difficult method, since the control points have to be determined from data points with the desired curve or surface accuracy and using iterative procedure. The least squares method is used to fit the data to curves and surfaces.

The problem of fitting NURBS curves using Least Squares method begins by considering the weights $w_i = 0$ or equal, as a way to have a linear problem and solve it to find the control points P_i. Given the data points $Q_0, \ldots, Q_m$ with different number of knots (m>n) and considering that $p \geq 1$ and $n \geq p$, equation 1 becomes as showed in equation 3, for $0 \leq u \leq 1$.

$$C(u) = \sum_{i=0}^{n} \left(N_{i,p}(u) \cdot P_i \right) \tag{3}$$

Considering that $Q_0 = C(0)$ and $Q_m = C(1)$, the remaining Q_k are determined minimizing the expression 4. The values $\bar{u}_k$ are the predetermined parameter values. The resulting curve does not pass exactly through Q_k data points. Setting the derivatives of function f with respect to the n-1 points P_l, equal to zero, it results in a system of n-1 equations with n-1 unknowns, showed in equation 5.

$$f = \sum_{k=1}^{m-1} \left| Q_k - C(\bar{u}_k) \right|^2 \tag{4}$$

$$\left(N^T \cdot N \right) \cdot P = R \tag{5}$$

The matrix N may be represented by expression 6 and the vector R with n-1 values is determined according to expressions 7 and 8 (k=1, … , n-1). The control points P is a column vector expressed as shown in equation 9 and to be determined.

$$N = \begin{bmatrix} N_{1,p}(\bar{u}_1) & \cdots & N_{n-1,p}(\bar{u}_1) \\ \vdots & \ddots & \vdots \\ N_{1,p}(\bar{u}_{m-1}) & \cdots & N_{n\_1,p}(\bar{u}_{m-1}) \end{bmatrix} \tag{6}$$

$$R = \begin{bmatrix} N_{1,p}(\bar{u}_1) \cdot R_1 + \cdots + N_{1,p}(\bar{u}_{m-1}) \cdot R_{m-1} \\ \vdots \\ N_{m-1,p}(\bar{u}_1) \cdot R_1 + \cdots + N_{n-1,p}(\bar{u}_{m-1}) \cdot R_{m-1} \end{bmatrix} \tag{7}$$

$$R_k = Q_k - N_{0,p}(\bar{u}_k) \cdot Q_0 - N_{n,p}(\bar{u}_k) \cdot Q_m \tag{8}$$

$$P = \begin{bmatrix} P_1 \\ \vdots \\ P_{n-1} \end{bmatrix} \tag{9}$$

The solution of these equations requires the knot vector $T = [u_0, \ldots, u_r]$ and the parameters $\bar{u}_k$. These parameters $\bar{u}_k$ may be calculated using expression 10, obtained for global interpolation, where d is the total chord length given by $d = \sum_{k=1}^{n} |Q_k - Q_{k-1}|$. The $\bar{u}_k$ values are expressed according to the equation 11 and the knot vector is composed by $T = \lfloor u_0, \ldots, u_p, u_{j+p}, \ldots, u_{m-p}, \ldots, u_m \rfloor$.

$$\bar{u}_k = \bar{u}_{k-1} + \frac{|Q_k - Q_{k-1}|}{d} \tag{10}$$

$$u_0 = ... = u_p = 0 \qquad u_{m-p} = ... = u_m = 1$$

$$u_{j+p} = \frac{1}{p} \sum_{i=j}^{j+p-1} \bar{u}_i \qquad j = 1,...,m-p \tag{11}$$

There is some commercial software dealing with NURBS determination, such as Catia, Matlab toolbox, Rhinoceros, Solidworks, among others. Considerations about the errors are presented as figures to evaluate the accuracy of generated surfaces.

3 Uncertainty determination and Monte Carlo Approach

The concept of measurement uncertainty is different from error definition and it was established by ISO Guide to the Expression of Uncertainty in Measurements (ISO GUM, 2008) as an international recommendation to express the results of any measurement. The uncertainty must be determined for all measurement results and its evaluation begins by establishing an equation or function relating all independent variables $(x_1, ... , x_n)$ that affect a given measurement system with the dependent variables (Y) under investigation ($Y=f(x_1,x_2,...,x_n)$). After that, the combined standard measurement uncertainty Uc_y is determined by the sum of the standard measurement uncertainties related to each x_1, x_2 ..., x_n variable multiplied by the respective partial derivatives of the function (f), plus a term expressing the correlation of any x_i and x_j variables with its correlated standard measurement uncertainty U_{xixj}, as showed in equation 12.

$$Uc_Y^2 = \sum_{i=1}^{N}\left[\frac{\partial f}{\partial x_i}\right]^2 \cdot U_{x_i}^2 + 2 \cdot \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \cdot \frac{\partial f}{\partial x_j} \cdot U_{x_i x_j} \tag{12}$$

The expression to determination of the combined standard measurement uncertainty may be rewritten in matrix notation to simplify the computational tasks (Cordero and Roth, 2005) as presented in equations 13 to 15. The vector S in equations 13 and 14 contains the partial derivatives called sensitivity coefficients. The matrix U_x in equation 15 is the covariance matrix that contains the uncertainties U_{xi} and the covariances U_{xixj}.

$$Uc_Y^2 = S \cdot U_x^2 \cdot S^t \tag{13}$$

$$S = \begin{bmatrix} \dfrac{\partial f}{\partial x_1} & \dfrac{\partial f}{\partial x_2} & \cdots & \dfrac{\partial f}{\partial x_n} \end{bmatrix} \tag{14}$$

$$U_x^2 = \begin{bmatrix} U_{x_1}^2 & U_{x_1 x_2} & \cdots & U_{x_1 x_n} \\ U_{x_1 x_2} & U_{x_2}^2 & \cdots & U_{x_2 x_n} \\ \vdots & \vdots & \ddots & \vdots \\ U_{x_1 x_n} & U_{x_2 x_n} & \cdots & U_{x_n}^2 \end{bmatrix} \tag{15}$$

Some important observations related to ISO GUM document are that it states that the following conditions are required to apply the methodology above: a) non-linearity of the function $f(x_1, ... , x_n)$ is insignificant; b) applicability of the Central Limit Theorem, with output variable represented by Gaussian probability density function; c) adaptation of the Welch-Satterthwaite formula to the determination of the effective degrees of freedom and then to calculate the expanded uncertainty as a multiple of the combined standard uncertainty Uc_y.

A more general approach was published as a supplement of the original ISO GUM document (ISO GUM Supplement, 2008). This supplement suggests the application of Monte Carlo simulation methods to determine the uncertainty through approximating a distribution function $G(\eta)$ to the value of the function $f(x_1, ... , x_n)$. Monte Carlo (MC) simulation approach for uncertainty determination considers that any value drawn at random from the distribution of input variables $(x_1, ... , x_n)$ is representative and valid as any other value of these variables. Then, it is possible to establish valid values to each input variable $x_1, ... , x_n$ based on its probability density functions (PDF) and thus determine the values and the distribution function of output variable Y. The figure 3 shows this approach.

The application of MC simulation is carried out with the following steps:

- generate a sample of size N by independently sampling at random from the PDF of each input variable $x_1, ... , x_n$;

when the input variables are dependent, it is required to use a joint PDF;

　　- for each sample, determine the corresponding value of output variable Y;

　　- repeat this procedure M times to yield M independent samples of size N and thus M estimated values of the output variable Y;

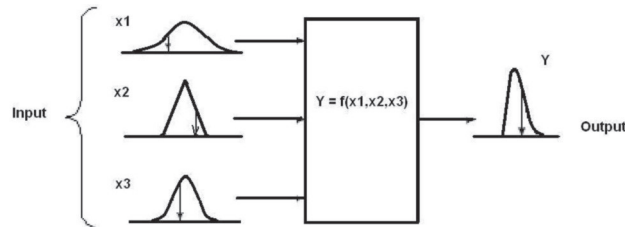　　- determine the average and standard deviation of the M trials.

Figure 2. Monte Carlo simulation approach to determine uncertainty.

When using Coordinate Measuring Machines (CMM), measurement of free form surface takes place by determining samples of points on the surfaces. These samples of points are then fitted using a defined algorithm to describe mathematically the desired geometry, profile or surface. There are some geometric elements like straight lines, circles and planes whose function are well known, but there is little information about uncertainty in estimation of the shape of free form surfaces and curves.

4 Experimental approach and results

The following steps were implemented: 1) capture coordinates of points on a free form surface using a CMM with rigid probe and save the data in the computer formatted as a text file (*txt*); 2) import data points (*txt* file) using MatLab software; 3) calculate of the knots vector; 4) calculate of the control points; 5) calculate the NURBS functions; 6) execute simulations after introducing the errors in NURBS parameters; 7) plot graphics of NURBS curves and surfaces.

The first step was performed by measuring curves on a turbine runner surface. Figure 3 shows the longitudinal lines were points were captured on the surface, corresponding to the measurement strategy adopted. A Coordinate Measuring Arm (CMA) Romer Arm 100 was used to carry out the measurements. This CMA has a rigid probe fixed at the arm extremity with a needle type stylus and it was operated by the software G-Pad. The CMA expanded uncertainty (95%) declared by the manufacturer was 0,086 mm to point-to-point length measurement and 0.07 mm to point coordinates. Two samples were determined with 25 and 100 points respectively and the point coordinates were saved in a *txt* file. Figure 3 shows the experimental setup with the turbine runner and the CMA.
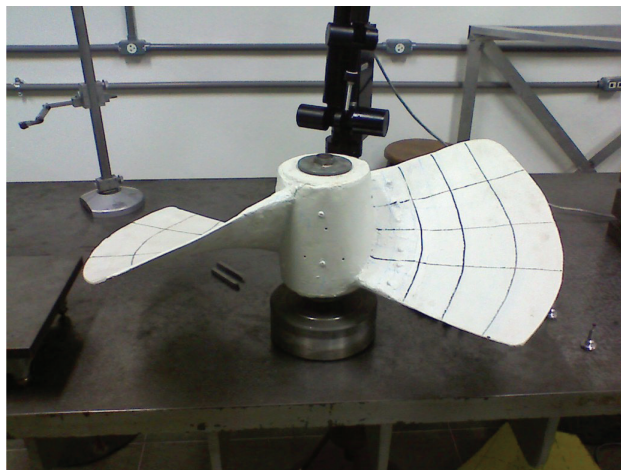

Figure 3. Experimental setup.

The calculations were performed using the MatLab software. The *txt* files were imported using the *dmlread* function and after this the knots vector was calculated using an algorithm developed with MatLab software. The control points were determined using this algorithm.

The basis functions were determined using an algorithm showed in figure 4. This algorithm was proposed by Piegl and Tiller (1997) to avoid divisions by zero. The input parameters required are the function degree p, the NURBS curve order m, the knots vector and the $\overline{u}_k$ values. At first, the algorithm verify the condition p=0, if the $\overline{u}_k$ value is within the interval [u_i, u_{i+1}] then $N_{i,0}(\overline{u}_k)$ becomes 1, on the contrary $N_{i,0}(\overline{u}_k)$ = 0. The p values are increased ($p \neq 0$) and calculations are performed.
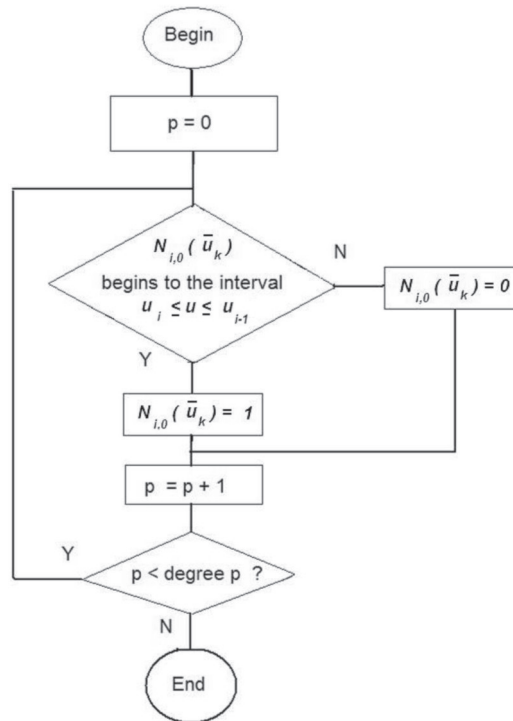

Figure 4. Simplified algorithm developed to determine the basis functions.

A NURBS curve was fitted to each group of points by using MatLab function *rsmak*. Figure 5 shows the fitted curve with the 25 measured points (blue) and 7 control points (red). It was observed a small difference between the data points and the fitted curve.

Simulation of deviations in data points were carried out to investigate variation in control points. Monte Carlo simulation was applied using MatLab software and the limits of variation of data points were determined by the measurement uncertainty declared by the CMA manufacturer. The maximum deviation imposed to coordinates x,y and z was 0.04 mm, estimated considering the CMA expanded uncertainty of 0.086 mm. The calculated control points are showed in figure 6 (right) with the determined NURBS curves and it was observed a deviation of about 2 mm between the data points (left) and the simulated curves at the xy projected plane.
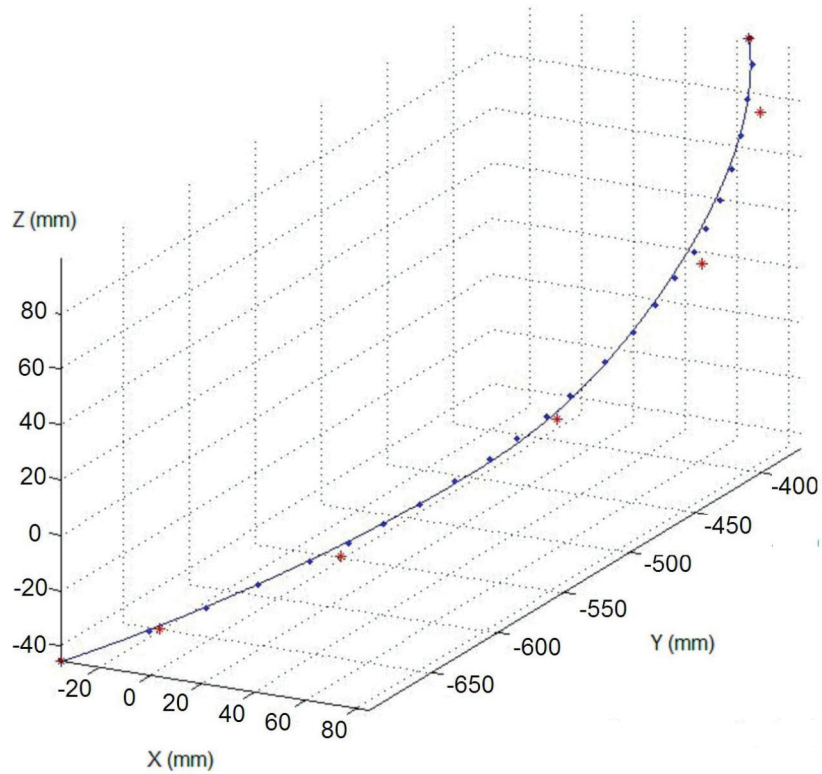
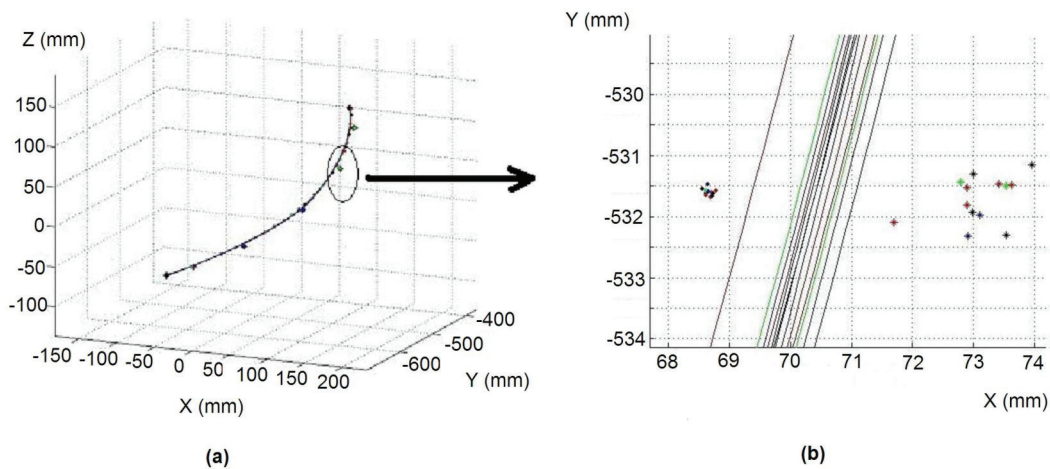**Figure 5.** Fitted NURBS curve with control and data points.

(a)

(b)

Figure 6. Simulated NURBS curves determined with simulated control points: (a) xyz view and (b) zoom at local region showed with xy view.

The knots vector was determined using these simulated data points. These values were used to perform simulation of variability of the knots and to determine the NURBS curves as showed in figure 7. Thirty curves were determined by simulation using a maximum deviation of 0.086 mm on the knots and it was observed deviations of about 5 mm between the curves and the point coordinates.

The number of control points was changed and simulation was applied to determine the NURBS curves. It was used 7 and 9 control points and the resultant NURBS curves are showed in figure 8. The curves are in agreement at the ends but there is a difference of about 8 mm near the center.
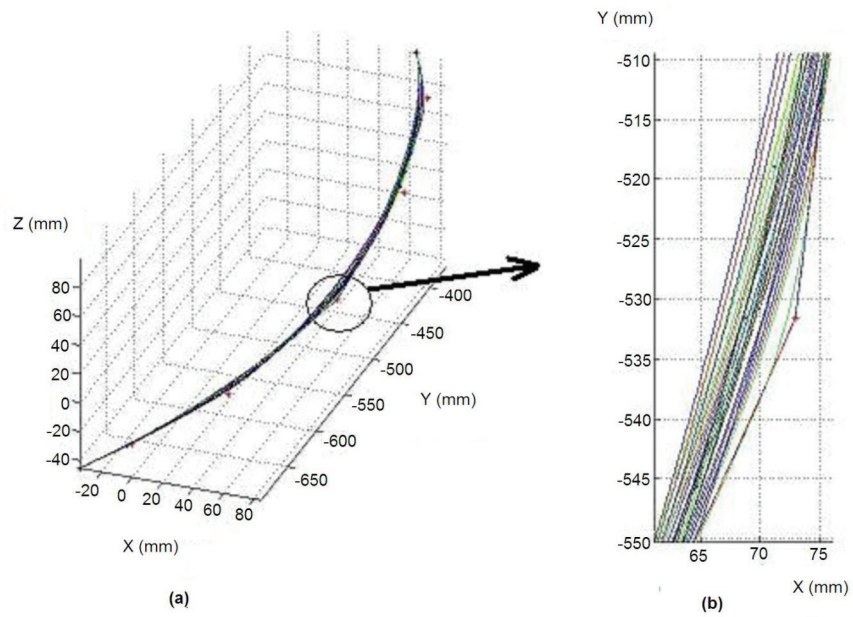
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Figure 7. (a) Curves simulated by changing knot vector; (b) curves with zoom at xy plane.
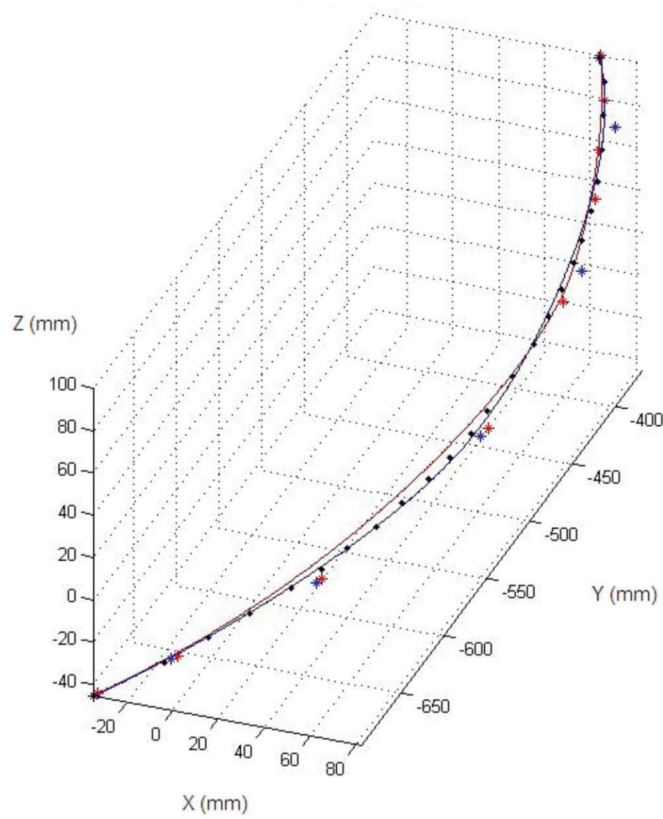

Figure 8. Curves with 7 and 9 control points.

5 Conclusions

The problem of fitting data to free form curves and the respective uncertainty determination was addressed and application of Monte Carlo simulation was proposed to generate uncertainty estimates of NURBS curves in agreement with the recommendations of the ISO Guide to the Expression of Uncertainty in Measurement, ISO GUM. Some tests were executed that showed the implications of changing the parameters of the NURBS curves like the knots vector, the control points and the number of control points over the spatial localization of the curves.

The preliminary results were obtained by measuring the profile of a turbine runner having nearly 300 mm long and it was observed a variation up to 8 mm in the curves localization when deviations of 0.04 mm were introduced in coordinates of data points. The effect of variation in control points was estimated 2 mm between the data points and the simulated curves at the xy plane. The effect of variation of the knots was estimated 5 mm and the effect of changing the number of control points showed the most pronounced as 8 mm was found.

The degree of difficulty of programming using NURBS models limited the extension of the simulation carried out. A more precise and quantitative estimate of the differences between data points and curves may be carried out by determining a set of 3rd degree curves joined together. As the NURBS curves parameters are closely related, it was observed that its determination requires human intervention to minimize the errors and the uncertainty in the fitted curves. Research is under development to apply the Monte Carlo simulation in its full definition and determine the uncertainty in measurement of free form surfaces.

Acknowledgements

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