

## Determining the preference threshold for a multicriteria decision support system through evolutionary modelling

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### Resumo

Este artigo usa os conceitos de Algoritmos Genéticos e de Algoritmos Transgenéticos associados a um sistema de apoio multicritério à decisão para alternativas discretas, visando definir melhor um importante parâmetro de ordenação, o limiar de preferência  $p$ . Este é um dos parâmetros essenciais para a aplicação dos métodos multicritério da Escola Francesa do Apoio Multicritério à Decisão que estão embutidos naquele sistema. Concluiu-se que a utilização de Algoritmos Genéticos pode ser extremamente útil na ajuda aos analistas de decisão, na seleção de valores para o limiar de transitividade. Recomenda-se, em pesquisas futuras, o estudo da possibilidade de se empregar Algoritmos Genéticos na análise de variações nos pesos dos critérios, bem como de variações nos valores das discordâncias. Além disto, deve-se também desenvolver pesquisas objetivando avaliar-se as pertinências dos pesos dos critérios, no contexto da modelagem através de conjuntos nebulosos, também embutida em Sistemas de Apoio à Decisão.

**Palavras-chaves:** Sistema de apoio multicritério à decisão. Algoritmos genéticos. Algoritmo evolucionário multiobjetivo

### Abstract

This article uses the concepts of Genetic Algorithms and Transgenetic Algorithms associated to a Multicriteria Decision Support System for discrete alternatives, with the aim of better defining an important parameter of ranking, in this case, the preference threshold,  $p$ . This is one of the crucial parameters for the application of multicriteria methods of the French School of Multicriteria Decision Support that are embedded in that system. The use of Genetic Algorithms was shown to be an extremely useful source for helping decision analysts in the choice of values for the thresholds of transitivity. For future studies, an investigation of the possibility of using Genetic Algorithms to study variations in criteria weights in detail and/or the value of discordances is recommended. In addition, studies could be carried out on the pertinences of criteria weights in the context of the use of this fuzzy parameter used by Decision Support Systems.

**Key-words:** Multicriteria decision support System. Genetic algorithms. Multiobjective evolutionary algorithm

## 1 Introdução

Genetic Algorithms (GAs) are metaheuristic, inspired by evolution and natural selection. GAs were introduced by John Holland in 1960 and are computational research methods based on genetic mechanism and natural evolution. In this way, they are used to solve search and optimization of adaptive form problems, based on the genetic process and the evolutionary process of living organisms. Populations composed of these organisms evolve according to the principles of natural selection and the survival of the fittest. GAs are generally efficient in the search for “good” solutions, according to the intervals or parameters considered. One of the advantages of their use is the simplification of the formulation and solution of optimization problems (Gandibleux & Ehrgott, 2005; Quintão, Nakamura & Mateus, 2005; Alencar, Villareal & Romero, 2005; Ramos et al, 2005).

Transgenetic Algorithms (TAs), in turn, are evolutionary algorithms which base their metaphor on a symbiogenetic process. Symbiogenetics is an evolutionary theory where individuals of different natures – that is, of different species -

unite to form a new individual. Symbiogenetics places greater emphasis on the positive effects resulting from genetic inter-relationships between individuals of different species rather than selection via reproduction of the fittest. Symbiogenetic relationships are more easily identified in the context of micro-organisms, but may also be the explanation for the greatest innovations of life, such as the eukaryotic cell and cellular metabolism (Michalewicz, 1996).

TAs use information from diverse sources and their process is strongly dependent on the quality of this information. It is also important for the information to be renewed during the procedure so as to prevent the premature convergence of the algorithm. (Van Veldhuizen & Lamont, 2000).

The symbiogenetic process of TAs is developed by the interaction of two populations of individuals of distinct species: chromosomes and transgenetic vectors. The interaction of the populations results in the forming of individuals fitter for environmental survival. In the evolutionary process, information is obtained both from the environment and from the evolutionary state of the populations. The interaction of the species occurs at three levels:

- The first contains the population of chromosomes which represent the memory of the result of the evolutionary process, in other words, the algorithmic search.
- The second level is composed of transgenetic vectors responsible for promoting the intensification and diversification of the population of chromosomes.
- The third level is formed by the rules which administrate the process of interaction between the populations of vectors and chromosomes, in this way characterizing a specific evolutionary scenario (Zitzler & Deb, 2000).

The symbiogenetic paradigm suggests that genetic or non-genetic information obtained a priori at the beginning of the evolutionary process can be used to guide the evolution, as the evolutionary past of the individuals is essential to enable evolutionary leaps. It is precisely in the union of the information contained in the individuals that the power for evolutionary acceleration resides. On the other hand, because it deals with an evolutionary process, information obtained during the process can also be used (Bäck, 1996).

As the transgenetic process belongs to the dimension of relationships between species, the exchange of information between individuals of the same species is an accessory element which can, in addition, simply not exist. In this way, the evolution of the population of chromosomes can occur without there being a direct exchange of information between their individuals. This exchange can be reflected in crossings or in the occurrence of mutations, a typical effect of the autonomous transformation of a population (Coello Coello, 1999).

The level associated with the rules for administrating the transgenetic model corresponds to the project and coordination of a process of inter-relationships between populations of individuals of different natures. However, one point is fundamental here: the species obtain and share information which increases their chances of survival, even though this in fact implies their radical transformation, leading to the creation of a new species (Coello Coello, 1999).

For symbiogenetics to function it is necessary for the species to be able to interact at a genetic level, even though they are of different species. In addition to this, each species must possess useful and complementary information. In the computational approach, the viability of genetic interaction is trivially guaranteed by the possibility of altering the configuration of the chromosomes and transgenetic vectors. On the other hand, it becomes essential for there to be compatibility among these interactions and that they have a common direction in the search for chromosomes (Deb, 2001).

This article uses the concepts of GAs and TAs associated to a Multicriteria Decision Support System (DSS), for discrete alternatives, with the purpose of defining an important ranking parameter, in this case, the preference threshold  $p$ . This is one of the crucial parameters for the application of multicriteria methods of the French School of Multicriteria Decision Support (Roy, 1995; Roy & Bouyssou, 1993), which are inserted in the DSS in question.

## **2 The multicriteria DSS**

The Multicriteria Decision Support System used in this study was THOR (an acronym for Algoritmo Híbrido de Apoio Multicritério à Decisão para Processos Decisórios com Alternativas Discretas - Multicriteria Decision Support Hybrid Algorithm for Decision Making Processes with Discrete Alternatives) which simultaneously aggregates the concepts of Rough Set Theory, Fuzzy Set Theory and Preference Theory (Gomes, 1999).

THOR is a Decision Support System for the multicriteria ranking of discrete alternatives, which eliminates redundant criteria simultaneously considering if the information is dubious – when using Rough Set Theory – and if there is an increase in imprecision in the decision process – in which case Fuzzy Set Theory is used. In this way, imprecision is

quantified, using it in the multicriteria decision support process. The concept of quantifying the imprecision associated with the weights and the classifications of the alternatives, put into operation in THOR, arises from the fact that the judgment values, because of their inherent subjectivity, cannot always be expressed in secure and precise ways. When using THOR, the simultaneous input of data into the process from multiple decision makers is also permitted, enabling these to express their judgment values in scales of ratios, intervals or ordinals, in addition to the execution of the decision making process without necessarily attributing weights to the criteria (Gomes, 1999).

The analytical modelling embedded in THOR is based on the ELECTRE methods of the French School of Multicriteria Decision Support (Roy, 1989; Roy & Bouyssou, 1993; Belton & Stewart, 2002). In this way, the following additional elements may be necessary for the application of THOR:

(i) a weight for each criterion, representing the relative importance among them;

(ii) a preference threshold (p) and another for indifference (q) for each criterion;

(iii) discordance;

(iv) pertinence of the values of the weights attributed to the criterion, as well as the pertinence of the classification of the alternative in the criterion.

Following the terminology of the School; with  $g(.)$  being a decision criterion; and  $a$  and  $b$  any two alternatives, the indifference threshold  $q$  is expressed by a function  $q[g(a)]$ , which can be constant in some situations, and represents an upper limit ( $q$ ) for the difference  $g(b)-g(a)$ ; any value for this difference less than  $q$  is not sufficient to guarantee strict preference, or even weak preference of  $b$  over  $a$ . The preference threshold, in turn, is represented by a function  $p[g(a)]$ , which represents the difference  $g(b)-g(a)$ ; it can be constant in some situations and represents a lower limit, below which there is not sufficient information to opt for a strict preference of  $b$  over  $a$ . Continuing to follow the terminology of the French School, discordance consists of the fact that there are no criteria in which the intensity of preference of  $b$  in relation to  $a$  goes beyond an acceptable limit (Vanderpooten, 1990; Roy, 1996).

It should be stressed that the relationships of outranking reached by means of THOR, instead of only indicating dominance, have a numerical quantitative which represents the “value of the alternative”, through an additive value function. This approximation permits the dominance relationship and the hierarchy of the values of the alternatives to be represented. Three situations are admitted for one alternative to be better than another:

- Situation 1 (or S1):

$$aPb \geq aQb + aIb + aRb + bQa + bPa$$

- Situation 2 or (S2):

$$aPb + aQb \geq aIb + aRb + bQa + bPa$$

- Situation 3 (or S3):

$$aPb + aQb + aIb \geq aRb + bQa + bPa$$

In those last three expressions the relationships of strict preference  $P$ , weak preference  $Q$  and indifference  $I$  are characterized in the formula below:

- $aPb \leftrightarrow g(a)-g(b) > + p$
- $aQb \leftrightarrow q < |g(a)-g(b)| \leq p$
- $aIb \leftrightarrow -q \leq g(a)-g(b) \leq + q$

Situation S1, only takes into account the alternatives  $a$  for which  $aPb$ , with  $b$  being any other alternative. In this way, comparing  $a$  with  $b$ , we can identify the criteria in which  $aPb$ , taking into consideration the thresholds of preference, indifference and discordance, checking if the condition imposed is satisfied. If satisfied, we know that  $a$  dominates  $b$ . Afterwards, the criteria weights in which this condition was met are added. For another alternative  $c$ , the same procedure described previously is repeated. The final scoring of alternative  $a$  will be the sum of the values obtained. For the situation S2, the alternatives for which  $aPb$  and  $aQb$  are taken into account. In situation S3, the alternatives for which  $aPb$ ,  $aQb$  and  $aIb$  are taken into account. It should be noted that the last two situations (S2 and S3) are less rigorous than the first (S1), so that a smaller difference permits one alternative to be classified better than another (Roy, & Bouyssou, 1993; Roy, 1996; Triantaphyllou, 2000).

The detailed description of the THOR algorithm and the software which supports it are in Gomes (1999; 2005).

### **3 Evolutionary modelling for determining p**

The dynamic of evolutionary modelling is relatively simple. Once an initial population is formed, the operations of crossing and mutation in individuals are carried out, prioritizing in some way the best adapted individuals, until a stop criterion is reached; at the end of the process, it is expected that the individuals have evolved to a good solution. It is important to analyse in what way some parameters influence the behaviour of the GAs so that it is possible to establish them according to the necessities of the problem and the resources available. There follow some comments on the principal parameters of the process:

- a) **Size of the Population:** the size of the population affects the global performance and the efficiency of the GAs. A “small” population offers a smaller coverage of the search space, causing a fall in performance. A “large” population supplies a better coverage of the domain of the problem and prevents premature convergence for local solutions. However, with a large population greater computational resources become necessary or a longer time processing the problem. In the face of this consideration, in the study approached in this article, an initial study population (sampling) was used with 560 alternatives (10% of the total alternatives) of a population of 5600 (this was the first generation). Afterwards, in the second generation, 1/3 of the alternatives were removed and an equal number included in their place, the new alternatives coming from the total population of 5600; another 1/3 of the original sample was “genetically” altered, this alteration occurred by alteration in the parameters of some alternatives (mutation) or junction of the parameters of the alternatives removed, creating new alternatives (crossing); and another third was not altered. In the third generation the best classified half in ranking was chosen from the first generation and the best classified half from the second generation (concept of elitism).
- b) **Crossing Rate:** the larger the rate, the faster new structures are introduced into the population.
- c) **Mutation Rate:** a low rate of mutation prevents a given position from stagnating at a value, as well as enabling any point to be reached in the search space. With a very high rate, the search becomes essentially random. The concepts of mutation and crossing were used together in the second generation when 1/3 of the alternatives initially used were altered.
- d) **Generation Interval:** controls the percentage of the population which will be substituted during the next generation. Two generation intervals were considered, as explained previously.
- e) **Substitution of the Population:** The majority of genetic algorithms make a generational type of substitution of the population, that is, all the elements of the current population are substituted by descendants. In some applications, the concept of elitism is used (applied in the third generation), that is, some of the solution proposals are preserved, those of the best quality, especially the incumbent. In the study presented here, the strategy of generational substitution of the population was not used. In this way, the proposal consisted of substituting some alternatives, creating mutations, making crossings, altering the population and conducting a new analysis, using the concept of elitism.

### **4 Genetic operators used**

The genetic operators transform the population through successive generations, extending the search until a satisfactory result is reached. For this purpose, it is necessary for the population to diversify and maintain adaptation characteristics acquired by previous generations. In this study only one generation was introduced.

The mutation operators are necessary for introducing and maintaining the genetic diversity of the population, arbitrarily altering one or more components of a chosen structure, in this way, supplying means for new elements to be introduced into the population. Mutation assures that the probability of reaching any point in the search space will never be zero. The mutation operator is thus applied to the individuals which, in this study, were alternatives. This application can be done in the following two ways:

- (i) Multi-point: is a generalization of the idea of exchanging genetic material through points, where as many crossing points as desired can be used.
- (ii) Uniform: does not use crossing points, but determines, through a global parameter, the probability of each variable being exchanged between the parents.

A multi-point concept was chosen for this study due to the fact that it is an explanatory study/work and consequently there is some difficulty in establishing the global parameter. The points used here were the classifications of the alternatives according to the multiple criteria, with the total number of these equal to 10 criteria.

Regarding the programming of genetic algorithms, there is an important aspect which differs a little from the habitual programming: it is more difficult to detect errors in the programming as the various lines of code will represent improvements on the initial scheme of evolution and, in many cases, the program will function and still supply apparently good results despite executing incorrect functions or ones that at least do not do what is expected of them. In the present study, though, particular attention was paid to this aspect, following the orientation of similar studies (Leyva-Lopez & Aguillera-Contreras, 2005).

### 5 Application of the new algorithm and its results

The purpose of the new algorithm used in this study was the maximizing of the thresholds p and q, so as to identify the relevant difference between the classifications of alternatives according to the multiple criteria. For this, the following problem was formulated and resolved:

Max (p, q) in:

- $S_1: aPb \geq aQb + aIb + aRb + bQa + bPa$
- $S_2: aPb + aQb \geq aIb + aRb + bQa + bPa$
- $S_3: aPb + aQb + aIb \geq aRb + bQa + bPa$

In those three last expressions we have:

Preference limit (p):  $aPb \leftrightarrow g(a) - g(b) > + p$

Indifference limit (q) :  $aIb \leftrightarrow -q \leq g(a) - g(b) \leq + q$

Situation of weak preference:  $aQb \leftrightarrow q < |g(a) - g(b)| \leq p$

Subject to:

$$p_i \geq q_i$$

$$p_i, q_i \in [0, 1], i = 1, \dots, 10.$$

Initially the total sampling of the alternatives in the criteria was checked, calculating the average of the classifications of the alternatives per criterion, afterwards calculating the standard deviation. The value of p was estimated as being 10% of the value of the standard deviation. The value of q was determined as being 10% of the standard deviation. In this way, Table I was obtained:

**Table I** : Criteria, weights and values of p

Designation of the Criteria	Criteria weights	Values of p
C	0.268618107	0.024
D	0.020431915	0.024
E	0.028089333	0.025
F	0.036145955	0.026
G	0.044772867	0.024
H	0.054255214	0.024
I	0.065106258	0.029
J	0.078352518	0.02
K	0.096389212	0.022
l	0.126402006	0.025

After this a sampling was made, composed of 560 alternatives from a total population of 5600 alternatives, the ranking of the 560 alternatives was obtained using the THOR Decision Support System. The alternatives were introduced through loading an Excel® spreadsheet. The computer used was a Pentium IV, with a 3.2 Ghz processor, with 512 M RAM memory. The time taken to load the spreadsheet in the THOR Decision Support System was 30 seconds with the time to generate each ranking 4 seconds, per ranking.

Proceeding in this way, the suggestions for the values of  $p$  were generated as shown in Table II. The first generation values were compared with the original values of  $p$ . The values suggested did not alter the original ranking of the alternatives.

**Table II:** Comparison between the original values and the values suggested for  $p$ , in the first generation

Original values of $p$	Values suggested in the first generation	Variations
0.024	0.097666	306.94%
0.024	0.097666	306.94%
0.025	0.098666	294.66%
0.026	0.099666	283.33%
0.024	0.097666	306.94%
0.024	0.097666	306.94%
0,029	0.102666	254.02%
0,02	0.093666	368.33%
0,022	0.095666	334.85%
0,025	0.098666	294.66%

It can be observed, from Table II, that the values of  $p$  were chosen in a very conservative way. Next, the alteration in the sampling was made as previously explained and the new values presented in the first column of Table III were obtained:

**Table III:** Comparison between the second and first generations of the values of  $p$

First generation	Second generation	Variations
0.097666	0.140623	43.98%
0.097666	0.140614	43.97%
0.098666	0.140622	42.52%
0.099666	0.140524	40.99%
0.097666	0.140724	44.09%
0.097666	0.140124	43.47%
0.102666	0.140122	36.48%
0.093666	0.140123	49.60%
0.095666	0.140524	46.89%
0.098666	0.140024	41.92%

Analysing Table III, it could be seen that the second generation had a comparatively smaller variation in relation to the first generation than the first generation had in relation to the initial values. It was decided to make a third generation, using the best classified half of each sample. Great care was taken not to repeat alternatives so that the study could continue with 560 alternatives. It was then observed that only 4 alternatives remained in the two samplings in the interval characterized as 50% higher, of each sampling, as shown in Table IV. The values obtained from the second generation were chosen for the value of  $p$ .



**Table IV :** Comparison of the second and third generations of the values of p

Third Generation	Second Generation	Variations
0.150508797	0.140623	7.03%
0.152425576	0.140614	8.40%
0.150535851	0.140622	7.05%
0.150459047	0.140524	7.07%
0.150616897	0.140724	7.03%
0.149974717	0.140124	7.03%
0.149426101	0.140122	6.64%
0.150450065	0.140123	7.37%
0.153030636	0.140524	8.90%
0.149755668	0.140024	6.95%

Alterations less than 8.91% were observed from the third to the second generation alterations. As a result, it was considered that there had been a convergence of the values.

## 6 Conclusion

The use of genetic algorithms has been shown to be an important source in helping decision analysts choose the values of the thresholds of transitivity. The analysis presented here suggested that, for the application of the THOR Decision Support System, the values of p and q be equal: the study started with pseudo-criteria, where p is equal to 10% of the standard deviation and 10% greater than q, both being different to zero.

The use of GAs assisted the decision agent in solving two problems in the selection of p:

- a) a value which is suitable to the study
- b) a value which remains suitable for future studies, for the same problem.

The suggestion was that quasi-criteria be adopted, with  $p = q$  and with these values greater than the original ones.

For future studies an investigation of the possibility of using Genetic Algorithms to study variations in criteria weights in detail and/or the value of discordances is recommended. In addition, studies could be carried out on the pertinences of criteria weights in the context of the use of the fuzzy parameter used by THOR. In future studies, if the situation  $p = q$  is judged to be inconvenient, a restriction of  $p \neq q$  could be included, plus  $p > q$ , instead of  $p_i \geq q_i$

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