CRITICAL STATE MODELING OF UNSATURATED SOILS: FINITE ELEMENT IMPLEMENTATION

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Abstract

This paper presents a critical state soil model used for the analysis of variations in water content associated with footing load. It is essentially a modified version of the Wheeler & Sivakumar (1995)'s model. An experimental validation of the model based on tests by Sivakumar (1993) is discussed here and studies a surface foundation lying on unsaturated soil subjected to water level fluctuations. Two hypothetical cases have been studied: a rise in the groundwater and a drop in suction from the ground surface, as in the case of rainfall or surface seepage from leaks in water pipes.

Keywords: *soil mechanics, unsaturated, elasto-plastic, finite elements*

Resumo

Este artigo apresenta um modelo de estados críticos para solos usado em análises com variação de umidade associado com o carregamento de uma sapata. É essencialmente uma versão modificada do modelo de Wheeler e Sivakumar (1995). São apresentados uma validação experimental de ensaios de Sivakumar (1993) e estudos de uma fundação assente numa camada de solo não-saturado, sujeita à variação do nível d'água. Dois casos hipotéticos são analisados: o levantamento do nível d'água e a diminuição da sucção a partir da superfície, como o caso de uma precipitação pluviométrica ou infiltração de tubulações enterradas.

Palavras-chave: *mecânica dos solos, solos não saturados, elasto-plástico, elementos finitos*

1 Introduction

The behavior of compacted kaolin under isotropic and triaxial tests conditions was predicted by WHEELER & SIVAKUMAR (1995) with a critical state model. This model used six parameters varying with suction [$\lambda(s)$, $N(s)$, $M(s)$, $\mu(s)$, $\psi(s)$ and $\Gamma(s)$] and three constants (k, k_s e G). The parameters were obtained from tests with four different suction values, considering a linear function between two consecutive values. This paper proposes some changes in the model of WHEELER & SIVAKUMAR (1995) by reducing the number of parameters, but with gain in model versatility. A fitting procedure is suggested to the variable suction parameters taken in tests with different suction values. In order to ensure consistency the model reduces to the modified Cam-clay when suction is zero. The proposed model also allows the use of data of the soil characteristic curves in a finite element program. A comparison between predictions using the proposed model and Wheeler & Sivakumar's model is presented against results of suction controlled triaxial tests performed by SIVAKUMAR (1993).

It is known in the literature (ALONSO, JOSA & GENS, 1993) that the collapse deformations by saturation increases with the stress state, then reaches a maximum value and then decreases with loading. This behavior may be predicted by elasto-plastic critical state models (WHEELER & SIVAKUMAR, 1995). The proposed model to be adopted here has the following features:

- Elasto-plastic rigidity $\lambda(s)$ may increase or decrease on a monotonic basis
- Strength parameters M(s) and $\mu(s)$ vary with suction
- The adjustment adopted for $\lambda(s)$ extends the stress range validity, in relation to the WHEELER & SIVAKUMAR (1995) model
- Decrease in the number of parameters in relation to the WHEELER & SIVAKUMAR (1995) model
- Considers the effect of the variation in water content on the specific weight of the material

The adopted model was installed in CRISP finite element program (BRITTO & GUNN, 1987) and validated against a number of practical situations (SILVA FILHO, 1998). The matrix equations used to implement the model in CRISP program are described below. This paper discusses the numerical simulation of a surface foundation on an unsaturated soil with variations in the water level. The present study considers both, the influence of the variation in water content on the specific weight of the foundation soil and the effect of submersion and the numerical simulation of a surface foundation on an unsaturated soil with variations in the water level. The present study considers both, the influence of the variation in water content on the specific weight of the foundation soil and the effect of submersion.

2 Model proposed by Silva Filho (1998)

The model proposed by SILVA FILHO (1998) adopts, for isotropic conditions, parameters adjusted from the experimental data of the isotropic consolidation lines and experimental LC surface. For triaxial conditions, the model reduces the number of parameters of the yield ellipse originally proposed by WHEELER & SIVAKUMAR (1995). However the slope of the critical state line M(s) and the intercept m(s) were still assumed to vary with suction.

2.1 Model's LC (Loading-Collapse) yield curve

To establish the p0(s) function, a fully elastic stress path is assumed, as shown in Figure 1. The stress path starts from the yield stress for saturated conditions $p_0(0)$, with increase in suction (line AB in Figure 1), followed by loading p (average stress less air pressure) under constant suction up to yield stress p0(s) (line BC in Figure 1). The value of the yield function p0(s) is based on the ABC stress path shown in Figure 1, using equation 1. This function describes an increase in the elastic behavior with suction.

$$p_0(s) = p^c \mathrm{EXP}\left[\frac{(\lambda(0)-\kappa(0))\ln\left(\frac{p_0(0)}{p^c}\right)+N(s)-N(0)+\kappa_s\ln\left(\frac{s+p_{atm}}{p_{atm}}\right)}{(\lambda(s)-\kappa(s))}\right] \qquad (1)$$

Where:

N(s) - specific volume referring to stress pc for suction s;
p^c - reference stress;
$\kappa(s)$ - slope of unloading and reloading line, for suction s;
$\lambda(s)$ - slope of virgin line for suction s;
$p_0(s)$ - isotropic yield stress for suction s;

The parameters varying with suction are obtained from isotropic consolidation tests, one of which is saturated, and two or more are unsaturated with constant suction values. The parameters obtained in the tests may however require adjustment to prevent inconsistent results for the range of stresses not used in the tests. The adjustment should be made for functions N(s) and $\lambda(s)$ based on the experimental LC and $\lambda(s)$ data. WHEELER & SIVAKUMAR (1995), suggested establishing the $\lambda(s)$ and N(s) values directly from the graph v : lnp. These parameters did not have pre-established functions as in other models (ALONSO, GENS & JOSA, 1990; FUTAI, 1997), and were obtained for discreet suction values, with a linear relationship adopted between two consecutive values.

This paper discusses a new adaptation of equation 1, which defines the LC flow surface. In this equation, the parameters varying with suction will be adjusted to extend the range of the model validity. Parameters N(s) and $\lambda(s)$ will be adjusted to two or more suction values, adopting a linear interpolation of the parameters between two consecutive suction values.

2.2 Parameters for isotropic stresses

The adjustments of parameters N(s), λ(s) and k(s) are given below, using the experiment values of the LC and isotropic compression lines.

Parameter N(s)

Parameter N(s) is adjusted using the experimental LC data. It can be calculated by obtaining the values of N(s) from the general LC curve, as in equation:

$$N(s)=N(0)+[\lambda(s)-\kappa(s)]\ln\left(\frac{p_0(s)}{p_{atm}}\right)-[\lambda(0)-\kappa(0)]\ln\left(\frac{p_0(0)}{p_{atm}}\right)-\kappa_s\ln\left(\frac{s+p_{atm}}{p_{atm}}\right) \qquad (2)$$

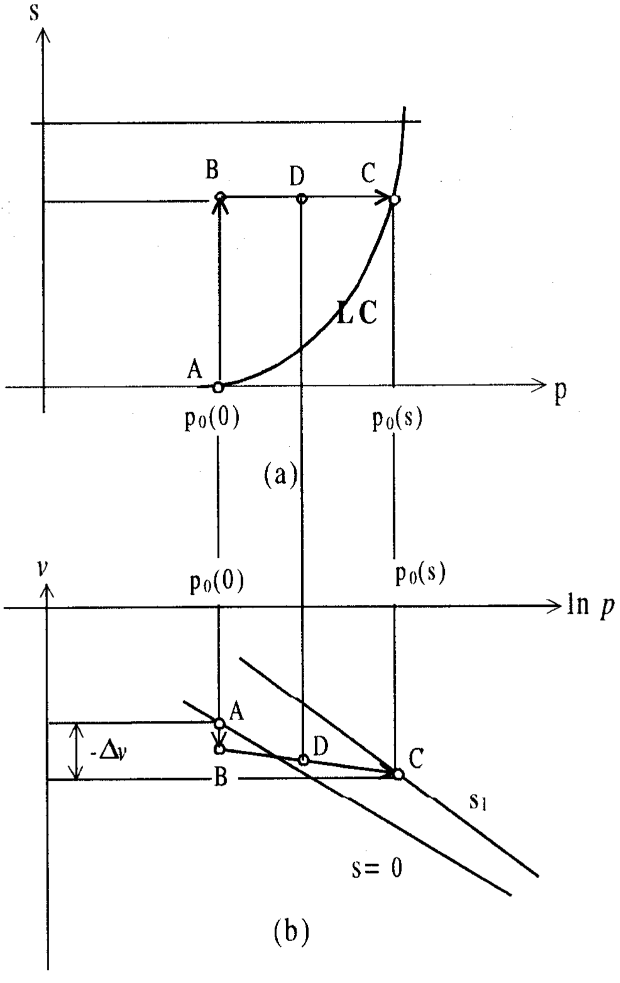

Figure 1 Establishing the equation of the LC flow curve

Parameter λ(s)

When using the λ(s) values directly from tests and extending the isotropic consolidation lines to different suction values, it is possible to cross these lines for high stresses. To prevent this inconsistency, the stresses are found in which each isotropic consolidation line intercepts the saturated line. These stresses will be referred to as $p_f(s)$, and correspond to the common point in the saturated isotropic consolidation lines, with suction equal to s, as in equation Parameter N(s)

$$p_f(s)=p_{atm}\exp\left[\frac{N(s)-N(0)}{\lambda(s)-\lambda(0)}\right] \qquad (3)$$

When $p_0(0) \geq p_r(s)$, parameters N(s) and $\lambda(s)$ will be equal to N(0) and $\lambda(0)$, respectively, so that from that point the consolidation line for suction s will be parallel to the saturated line, as shown in Figure 2.

Parameter k(s)

DELAGE & GRAHAM (1996) point out that suction controlled isotropic tests from different authors in which k(s) was not significantly affected by suction. CUI & DELAGE (1993) showed isotropic tests with loading at suctions varying from 400 to 1500 kPa and the slope of the unloading and reloading curves for different suctions, are quite similar. Based on those results, the decision was to opt for the constant parameter k, regardless of suction.

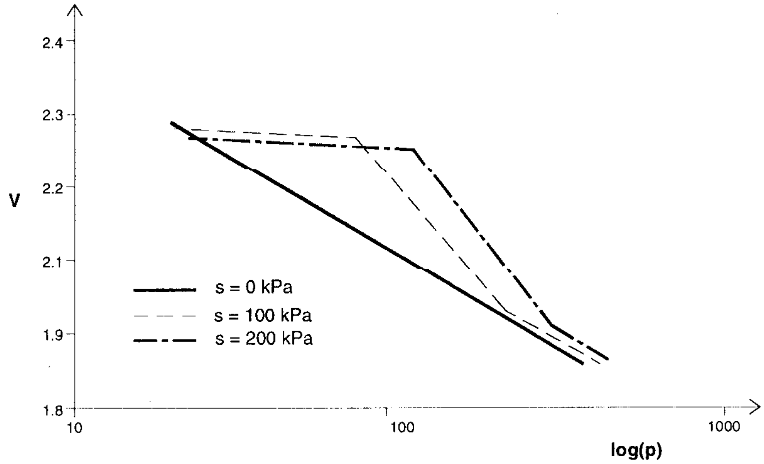

Figure 2 Isotropic compression curves to prevent crossing

2.3 Proposed model for triaxial stress states

The use of the WHEELER & SIVAKUMAR (1995) model for the condition $q \neq 0$ requires more parameters depending on suction. On the plane (p,q), WHEELER & SIVAKUMAR (1995) suggested an elliptic yield curve differing from that proposed by ALONSO, GENS & JOSA (1990). The equation of the ellipse adopted in the proposed is different from that suggested by WHEELER & SIVAKUMAR (1995) and is given below.

$$q^2 = M(s)^2 (p + \mu(s))(p_0(s) + p) \tag{4}$$

In equation 4 the parameter $\mu(s)$ does not have the same meaning as that proposed by WHEELER & SIVAKUMAR (1995). In the proposed model, the flow ellipse touches the axis of stress p at two points: $p_0(s)$ and $\mu(s)$, as in Figure 3. The parameter $\mu(s)$ may vary linearly or non-linearly with the suction and given as:

$$\mu(s) = \frac{\overline{\mu(s)}}{M(s)} \tag{5}$$

Where:

$\overline{\mu(s)}$ - Interceptor of the critical state line with axis q;

M(s) - slope of the critical state line, varying with suction.

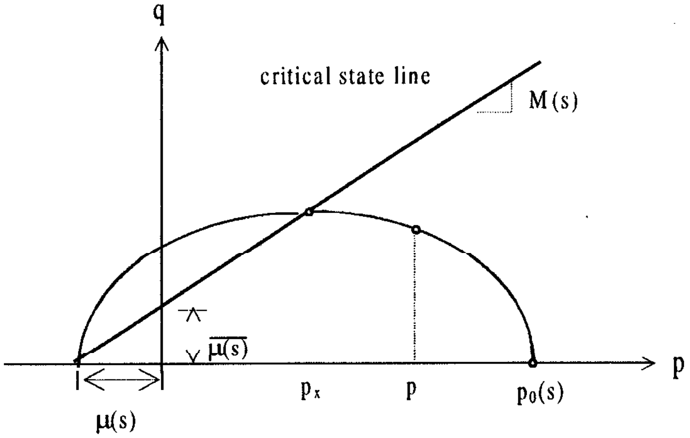

Figure 3 Parameters for triaxial stress state

2.4 Change in specific weight

The change in water content, due to changes in soil suction, often causes a significant variation in the soil specific weight. This effect may be considered in the proposed model from the data of suction against volumetric water content θ. Such a relationship, which is named the soil's characteristic curve, is shown in Figure 4. This curve presents hysteresis, i.e., it differs depending whether it consists of the drying or wetting trajectories. For example, curve 1 in Figure 4 corresponds to the drying and curve 2 to the wetting of the soil. In order to consider the influence of the water content in a finite element code, a vector of nodal forces responsible for changes in specific weight was included in the computational model, as described below.

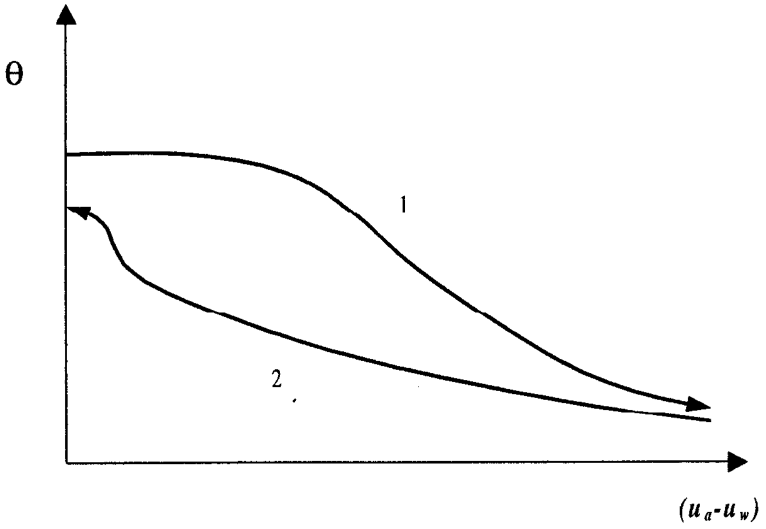

Figure 4 Soil characteristic curve with hysteresis effect.

2.5 Model's Advantages

It is known in the literature that the collapse deformations by saturation increases with the stress state, then reaches a maximum value and then decreases with loading. This behavior may be predicted using the elasto-plastic critical state models proposed by WHEELER & SIVAKUMAR (1995) and FUTAI (1997). The proposed model offers further advantages, as follows:

- Elasto-plastic rigidity λ(s) may increase or decrease on a monotonic basis or not
- Strength parameters M(s) and μ(s) vary with suction

- The adjustment adopted for $\lambda(s)$ extends the stress range validity, in relation to the WHEELER & SIVAKUMAR (1995) model
- Decrease in the number of parameters in relation to the WHEELER & SIVAKUMAR (1995) model
- Considers the effect of the variation in water content on the specific weight of the material

3 Deployment of the model in a finite element program

The general elasto-plastic stress-strain relationship for critical state models for unsaturated soils that is given by (SILVA FILHO, 1998)

$$d\sigma' = D_{ep}\left\{ d\varepsilon - \left(m\frac{\kappa_s}{3v(s+p_{at})} + \frac{\dfrac{\partial Q}{\partial \sigma'}\dfrac{\partial F}{\partial s}}{-\dfrac{\partial F}{\partial \varepsilon_v^p}\dfrac{\partial Q}{\partial p'}} \right) ds \right\} \tag{6}$$

and

$$m = \{1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0\}^T$$

Where:

D_{ep} - elasto-plastic matrix;
D_e - elastic matrix;
$d\varepsilon$ - increase in total specific deformation;
k_s - compressibility coefficient throughout the variation trajectories of s in unloading and reloading;
v - specific volume;
s - suction;
p_{at} - atmospheric pressure;
Q - plastic potential;
F - flow function;
σ' - normal stress;
p' - average stress;
ε_v^p - specific plastic volumetric deformation.

$$D_{ep} = D_e - \frac{D_e \dfrac{\partial Q}{\partial \sigma'}\left\{\dfrac{\partial F}{\partial \sigma'}\right\}^T D_e}{-\dfrac{\partial F}{\partial \varepsilon_v^p}\dfrac{\partial Q}{\partial p'} + \left\{\dfrac{\partial F}{\partial \sigma'}\right\}^T D_e \dfrac{\partial Q}{\partial \sigma'}} \tag{7}$$

For stress paths involving a variation in suction in a finite element program, it is necessary to deploy a suction variation loading vector, which enters the stress-deformation relation through the aforementioned initial deformation technique. This vector is as follows:

$$\int_{V^e} B^T D_{ep} \varepsilon_0 d(vol) \tag{8}$$

where:

B - deformation-displacement matrix;

$$\varepsilon_0 = m\frac{\kappa_s}{3v(s+p_{at})}ds + \frac{\dfrac{\partial Q}{\partial \sigma'}\dfrac{\partial F}{\partial s}}{-\dfrac{\partial F}{\partial \varepsilon_v^p}\dfrac{\partial Q}{\partial p'}}ds \tag{9}$$

The vector of variation of specific weight with water content, a consequence of the actual suction variation is given as:

$$\int B_i^{(e)^T} \delta\theta\gamma_w{}^{(e)} dV^{(e)} \qquad (10)$$

Where:

$\delta\theta$ - variation in water content;

γ_w - specific weight of water.

4 Validation of the model: tests performed by Sivakumar (1993)

Comparisons of predictions between the proposed model and Wheeler & Sivakumar's model will be presented in this item. The data of suction controlled triaxial tests performed by SIVAKUMAR (1993) in statically compacted kaolin samples will be used for this comparison.

4.1. Tests performed by Sivakumar (1993)

The tests by SIVAKUMAR (1993), consisted of three phases: (a) equalization of water and air pressures; (b) isotropic consolidation; (c) shear in a triaxial cell. The soil was compacted in nine layers with water content 5% below the optimum water content for the Proctor test.

In the equalization stage, some samples were taken to a hydrostatic stress p = 50 kPa and suctions of 100 kPa, 200 kPa or 300 kPa. Eight samples were compacted and saturated with average stress p = 25kPa at the end of the equalization process. After the equalizing stage, the samples were loaded until hydrostatic p = 100, 150, 200, 250 and 300 kPa. After consolidation, the samples underwent shear in different conditions. Only the conditions to be adopted in the predictions using the proposed model will be described.

- Type (B) tests – shear test with constant stress p and suction. The increase in u_a- and u_w to the same proportion was also used in these tests, keeping both the suction and stress p constant;
- Type (C) tests – drained shear test with constant suction. For this test, the pressures u_a- and u_w were kept constant during shearing;

4.2. Predictions of tests by Sivakumar (1993)

The parameters adopted in the proposed model, based on tests by SIVAKUMAR (1993), are presented in Table 1. The initial flow curve is located at $p_0(0)$ = 18 kPa. The same parameters were adopted in WHEELER & SIVAKUMAR (1995) model, but these authors adopted 2 additional parameters. Some type B and C tests were chosen at random to be use for the predictions in the shear stage. The initial state of stress and end of each test run are given in Table 2.

Table 1 Parameters variable with suction adopted in the present model

Suction (kPa)	$\lambda(s)$	$N(s)$	$\mu(s)$	$M(s)$	$\kappa(s)$	$p_f(s)$ (kPa)
0	0.128	2.052	0.0	0.813	0.02	
100	0.182	2.122	54.2	0.933	0.02	303.4
200	0.196	2.196	83.5	0.959	0.02	649.0
300	0.176	2.212	122.0	0.910	0.02	1656.0

Table 2 Stress states before and after the shear stage (SIVAKUMAR, 1993)

Test	Before shear			After shear		
	p (kPa)	Q (kPa)	s (kPa)	p (kPa)	q (kPa)	s (kPa)
7B	200	0	200	198.6	264.6	200
8C	150	0	200	262.5	337.5	200
11B	100	0	100	100	145	100
18C	150	0	300	271	364.8	300

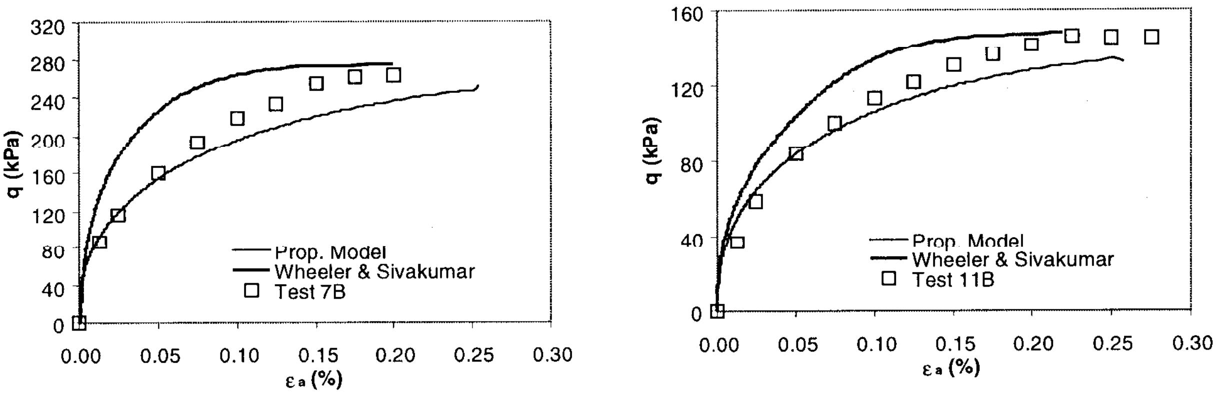

Figure 5 Prediction of tests by SIVAKUMAR, 1993 (Test 7B, 11B)

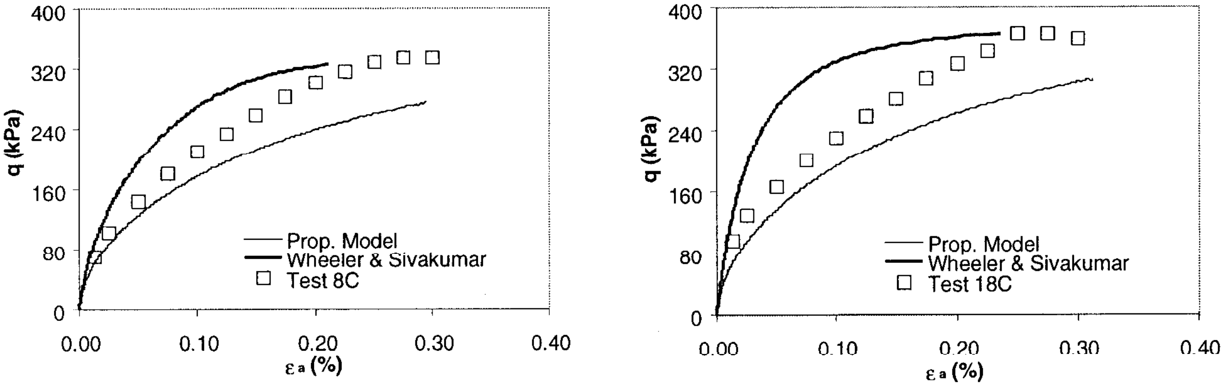

Figure 6 Prediction of tests by SIVAKUMAR, 1993 (Test 8C, 18C)

The results of the predictions using the WHEELER & SIVAKUMAR (1995) models and the modified proposal can be seen in Figure 5 and 6, compared with the test results. It is noticed that the adjustment with the proposed model is better for low stress-strain levels.

The critical state condition in almost every test was better predicted with the model by WHEELER & SIVAKUMAR (1995). However it can be argued that the proposed model uses less parameters and that the usual stress range in many practical situations is far from the critical state. Therefore, it may be more advantageous to use the model presented in the paper herein.

5 Finite element analysis using the proposed model

5.1 Surface foundation on collapsible soil

NESNAS (1995) discussed a hypothetical footing on an unsaturated soil layer. In this hypothetical situation, the specific weight of the soil was assumed to be equal to 20 kN/m^3, regardless of the water content in which it was found, and a total thickness of 12m with the water level 3m below ground level. Due to erosion, 1m of this soil was removed resulting in a total layer with a depth of 11m. As a result of seasonal fluctuations, the water level then dropped three meters from its initial position, as shown in Figure 7.

The footing was built 2m below ground level, corresponding to a distributed surcharge of 40kPa, and the soil in this configuration is saturated at depths bellow 3m. The removal of 1m of soil reduced the vertical effective stresses, bringing the soil in this zone to the temporarily over-consolidated condition. The subsequent 3m lowering of the water level increased the vertical effective stresses by 30kPa, causing the soil to return to the normally consolidated state.

For depths 3m below the foundation level, the soil was saturated at the time when the 1m-overload layer was removed, but became unsaturated as the water level lowered. The removal of the overload resulted in a 20kPa drop in vertical effective stresses, so that the soil in this zone became over-consolidated.

For the soil layer between 2m and 3m below the foundation's settlement level, the groundwater lowering resulted in an increase of 20 to 30 kPa in the vertical effective stress. This zone was in an over-consolidated state immediately before the de-saturation, returning to the normally consolidated conditions, since the increase in suction increases the yield stress, for the final suction value. In the situation described herein, NESNAS (1995) simulated the loading of the foundation equal to 30kPa and raising the groundwater using the model by ALONSO, GENS & JOSA (1990).

The present paper introduces the hypothesis of the variation in water content influencing the soil stress state, which is possible using the proposed model (SILVA FILHO, 1998). As the groundwater rises, there is an increase in the water content and consequently an increase in the effective specific weight of soil (γ'), until immediately before saturation. Considering the increase in the volumetric water content, the foundation soil shows a drop in volume caused by the increase of γ'.

After saturation and the increase in pore pressure the inverse occurs, that is, the relief of effective stresses causes an increase in volume of the mass. The simulation of suction reduction caused by rainfall has also been carried out and is shown below.

The model parameters used in the present analyses were obtained from the tests by JOSA (1988), and are shown in Table 3. The last column in this table corresponds to hypothetical volumetric values of water content, estimated based on the soil's suction.

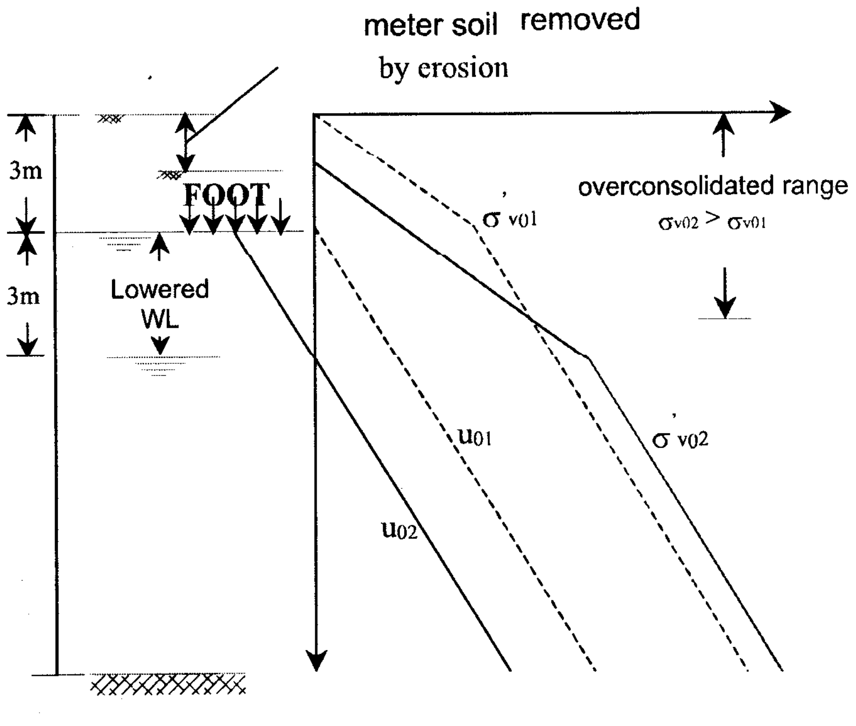

Figure 7 Distribution of vertical stresses and pore-pressures

Table 3 Parameters variable with suction (Proposed Model)

Suction	$\lambda(s)$	$N(s)$	$\mu(s)$	$M(s)$	$\theta(s)$
0.	0.14	2.125	0	0.82	0.45
15.	0.126	2.137	22.7	0.82	0.15
30.	0.113	2.145	45.3	0.82	0.08
45.	0.103	2.154	68.1	0.82	0.05

Figure 8 presents the finite element mesh adopted and the results in terms of displacement vectors. As the soil below the groundwater is normally consolidated, the application of the footing loading caused the soil yield, thus the compression of the whole soil mass, showing the vertical displacements downwards, as shown in Figure 8.

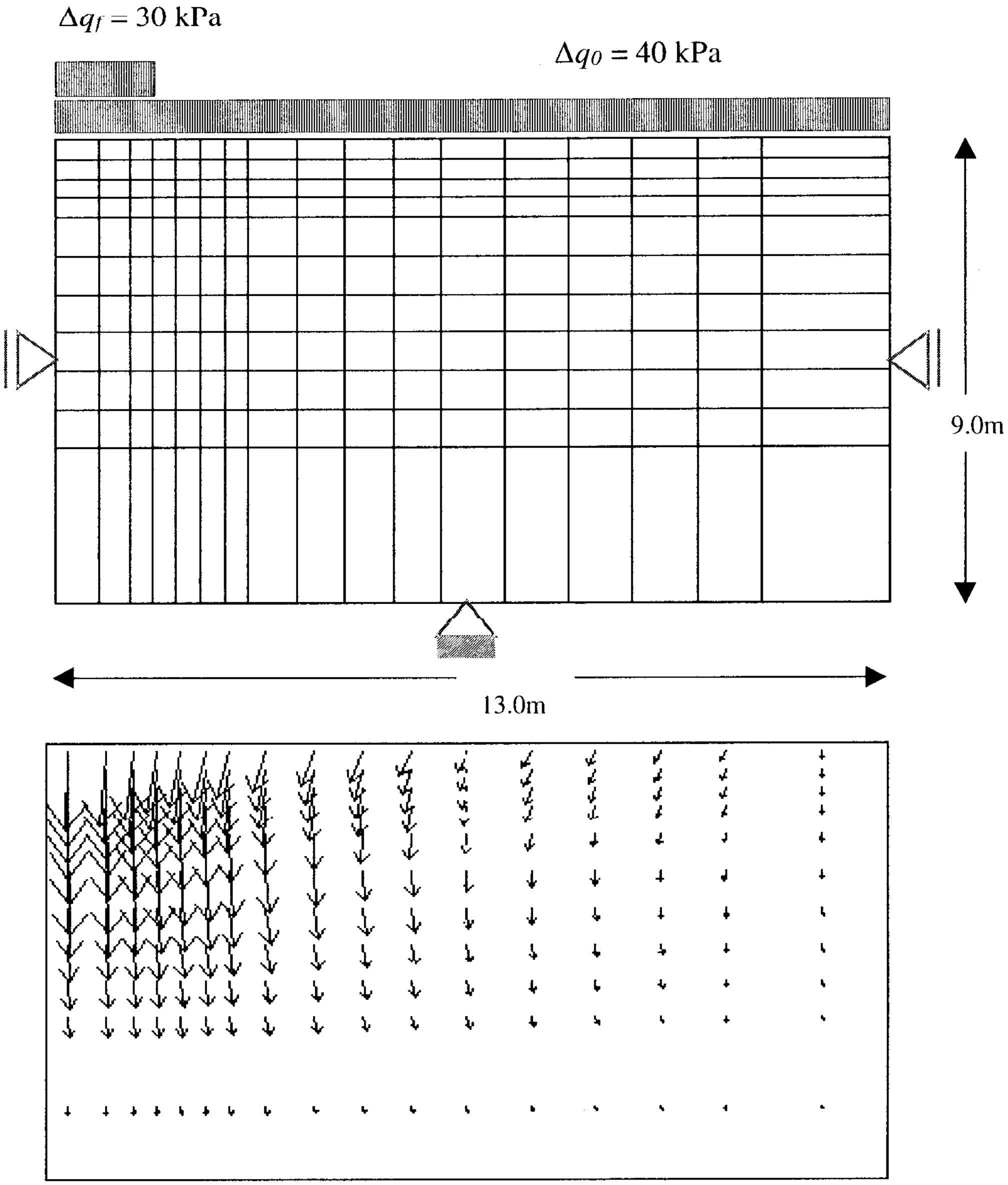

Figure 8 Vertical displacements after footing loading (vectors increased 200 fold)

5.2 Raising of the WL in 1.5m

The final position of the groundwater and new pore pressure distribution can be seen in Figure 9. During the raising of the groundwater to 1.5m below the settlement level of the foundation, there was a collapse in the stretch near the foundation, since the stresses applied activated the yield stress for saturated conditions $p_0(0)$ at several points. In the zone farthest from the loading, the suction decrease was smaller, thus occurred inside the elastic region with little expansion. Figure 9 shows the vertical displacement at the footing's settlement elevation.

This slight expansion corresponded to the relief of effective stresses, caused by the saturation of the region between 3.0m and 1.5m below the footing. Both the drop in effective stresses, and the reduction in suction, expanded the soil above the initial water level, which was slightly pre-consolidated.

A second case just differs from the first case in the increase in the water content and consequent variation in the specific weight of the soil γ. Therefore, as the variation in the water level increases, the load vector in the finite element solution shall consider the suction variation, also with a possible pore pressure variation, for each integration point, and an apparent increase in the specific weight as the water content increases.

The vector of nodal forces applied to the element, in this situation, may have up to three portions and the end result may be compression or expansion. It is possible in the same element to have connection points that vary only in suction and water content, and others where the water level variation saturates and thus the integration point will also have to consider the positive variation of the pore pressure of the water u_w.

If the variation in the suction is on an elastic basis, its unloading and increase in pore pressure, with less effective stresses, tend to expand the soil, while the apparent increase in its specific weight tends to compress it. If the variation in the suction behaves in an elasto-plastic manner, its unloading will lead to the collapse of the soil, with a consequent reduction in its volume. The other two portions will behave in the same way.

WHEELER (1996) points out the different behavior of the variation in the water content when there is less elastic or elasto-plastic suction. WHEELER (1996) criticizes the formulations that do not consider this fact, but instead are chiefly concerned with loading with the constant gravimetric water content and which causes a variation in the suction. The situation discussed in this paper refers only to the increase in specific weight with the volumetric water content, which, although simplified, does not incur conceptual errors like those queried by WHEELER (1996).

Figure 10 shows the vertical displacements at the footing's settlement level, calculated numerically, after the raise in the water level, considering the increase in water content. It is found in this situation that the end displacements in the mesh that showed an expansion are, in this case, smaller than those when the variation in γ was not considered. When the increase in the water content was also considered, the collapse was on a larger scale, as shown in Figure 9.

5.3 Linear decrease in suction above WL

A situation that may occur in practice is the decrease of suction from ground level. This is possible due to surface seepage through leaks from existing pipes close to the work site, or even rainfall. In such a situation, it is necessary to establish how the suction will vary. This will depend on the intensity of the seepage flow and type of soil, but very often the suction will not be reduced to zero.

Figure 11 shows the pore pressure distribution adopted in this example. It is possible to assume the suctions also varying horizontally, due to different drainage conditions, but this was not assumed here.

The results of displacements corresponding to this suction variation are shown In Figure 12. The results are similar to the raise in water level. However, for points away from the foundation, the displacements are close to those from the simple footing load. The displacements at these points for the previous analysis (raise in water level) are bigger due to full saturation within 1.5m. The consideration of the increase in specific weight of the soil with the variation in water content caused larger displacements, as shown in Figure 12. The compression in the region away from the footing was the result of the consideration of the increase of γ with the increased water content.

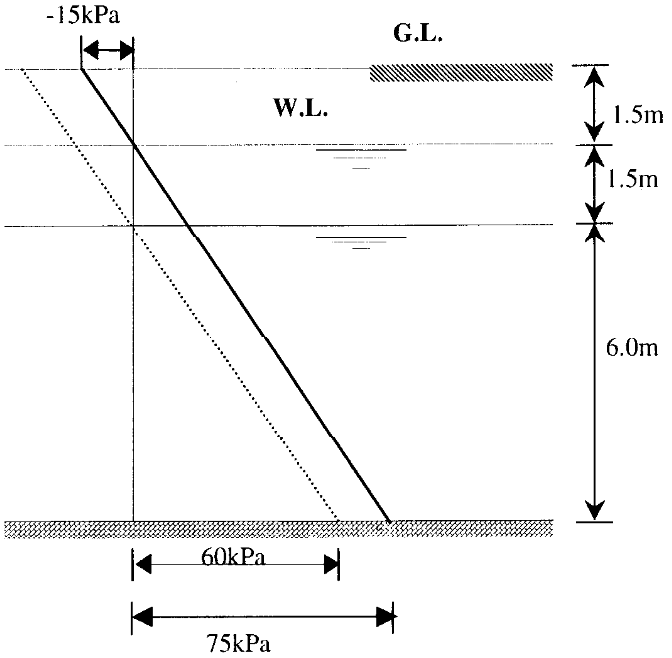

Figure 9 Distribution of pore pressures showing the 1.5m rise in the water level

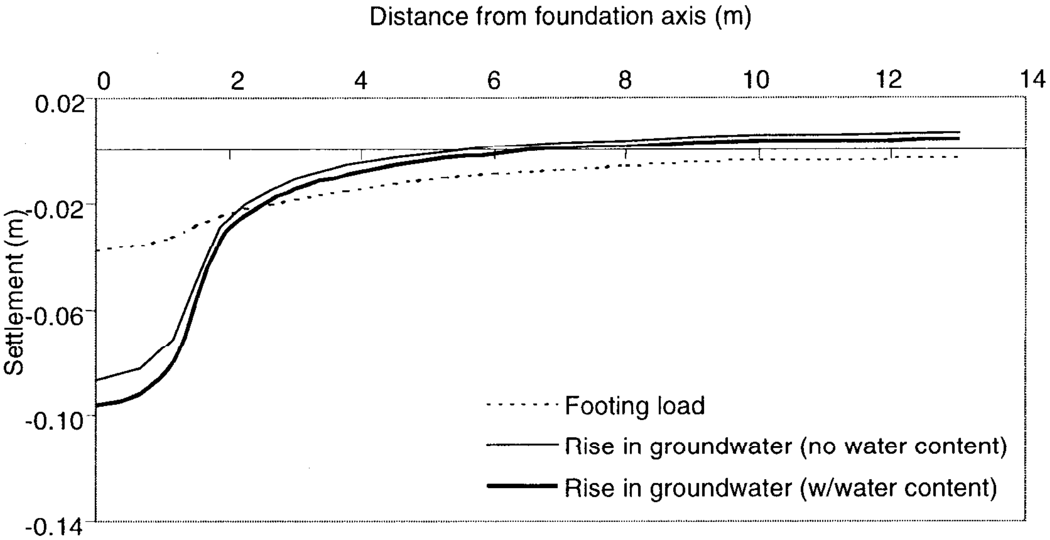

Figure 10 Vertical displacements calculated at the footing level

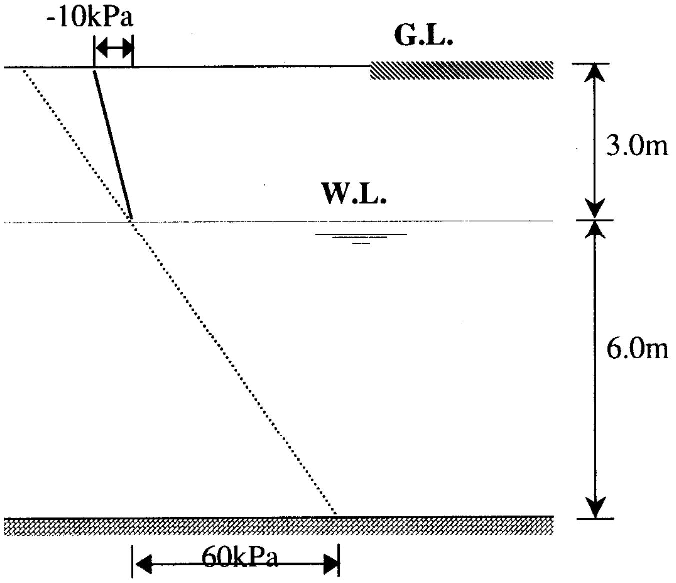

Figure 11 Distribution of pore pressures for reduced suction at the ground surface

6 Conclusion

This paper presents a critical state model for unsaturated soils proposed by SILVA FILHO (1998). The model handles the elasto-plastic rigidity in such a way that there may be increasing or decreasing behavior with suction. The parameters of the critical state line also vary with the suction. The fewer parameters in relation to the Wheeler & Sivakumar's model and the type of adjustment adopted for the compressibility are other advantages of the proposed model. SIVAKUMAR (1993) tests were used to validate the model, comparing the results of the predictions with the proposal of WHEELER & SIVAKUMAR (1995). It may be concluded that, the predictions with the proposed model are slightly better than the Wheeler & Sivakumar's model, particularly for stress levels away from the critical state. The consideration of the effect of the variation in water content on the specific weight of the material is also other model advantage. Considering that the proposed model uses less parameters and that in many practical situations the usual stress band is very far from the critical state, it may be concluded that the model is a good option for stress-deformation analysis of the unsaturated soils, particularly when collapsible behaviour is involved.

The influence of the variation in water level on a soil profile subject to a footing load has been analyzed with a new model implemented in CRISP finite element program. It was shown that the rise in water level results in collapse under the footing and expansion away from the footing. A different situation was also analyzed in which the suction is reduced linearly from the ground surface to the water level, simulating a rainfall, for instance, resulting in just collapse. It may also be found that the soil behaves differently in the studies in which is considered the increase in the specific weight of the soil with the water content and when this is not taken into account.

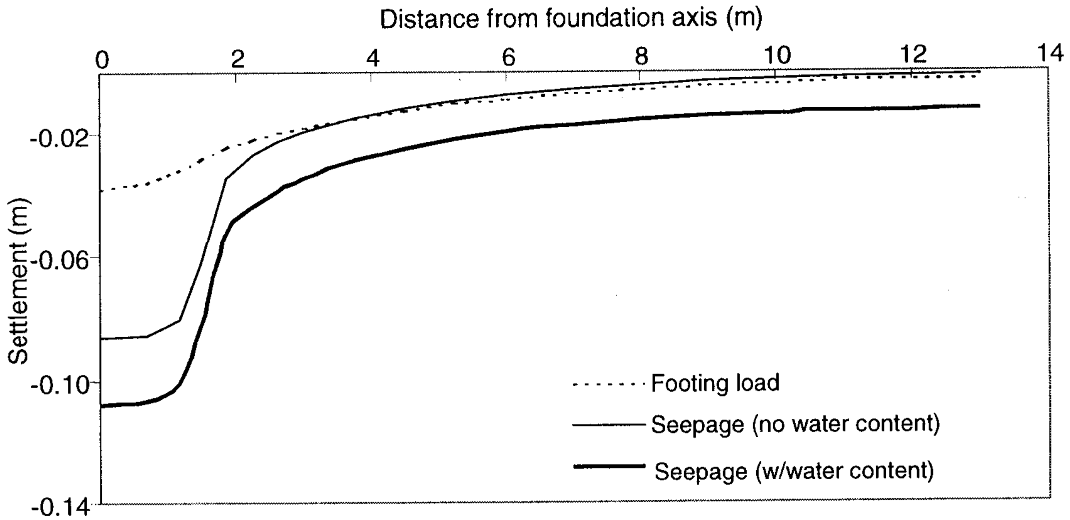

Figure 12 Vertical displacements calculated at the footing level

7 References

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