

## THE EFFECT OF PRESSURE IN 2D LUIKOV DRYING PROBLEM

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### Resumo

O processo de secagem em um meio multidimensional capilar poroso, neste caso uma cápsula de epóxi, usado em revestimento de componentes eletrônicos, é visualizado através do modelo de Luikov, no qual os potenciais de temperatura, umidade e pressão são solucionados analiticamente utilizando-se técnica de transformada integral generalizada (GITT), aplicando-se um controle de erro global sobre a solução obtida. O comportamento da convergência numérica e do próprio processo de secagem é descrito através de gráficos e tabelas.

*Palavras-chave: método da transformada integral, transferência simultânea de calor e massa, meios porosos, termodinâmica, problema de secagem de Luikov.*

### Abstract

Multidimensional drying in capillary porous media is analytically solved for the associated temperature, moisture and pressure content distributions. Luikov's model with linear transport coefficients and two-dimensional plate geometry is adopted for description of the simultaneous heat, mass and pressure transfer phenomena. The generalized integral transform technique (G.I.T.T.) is applied to the problem and the automatically global error-controlled solution of the coupled partial differential equations is used to achieve the solutions. The convergence behavior of the proposed eigenfunction expansions is illustrated.

*Keywords: integral transform method, simultaneous heat and mass transfer, porous media, thermophysics, drying, Luikov problem.*

## 1 Introduction

The system of equations proposed by LUIKOV (1975) is by far the most frequently adopted in the study of drying phenomena in capillary porous media with various applications in the engineering and applied sciences. The integral transform method (COTTA, 1993; COTTA & MIKHAILOV, 1997) has been successfully utilized in the hybrid numerical-analytical solution of such problems, for both the linear (DUARTE, 1995, 1998; RIBEIRO, COTTA & MIKHAILOV 1993) and non-linear versions (RIBEIRO & COTTA, 1995; DUARTE, 1998), offering the attractive feature of automatic global error control in the final results. Both applications previously considered (DUARTE, 1995, 1998; RIBEIRO, COTTA & MIKHAILOV, 1993; RIBEIRO & COTTA, 1995) where the interest in studying multidimensional situations are ever increasing, as demonstrated by the finite element method numerical solution in FERGUSON, LEWIS & TOMOSY (1993). Therefore, the present contribution advances the integral transform methodology to be applicable in multidimensional drying problems, such as the one formulated in LEWIS et al. (1996) and THOMAS, MORGAN & LEWIS (1980), and demonstrates another attractive feature of this class of hybrid method, i.e., the just very mild increase in computational effort for increased number of dimensions in the problem (independent variables). Essentially, it is reconfirmed that the overall computational cost in implementing the one-dimensional simulation is exactly comparable to that of solving the two-dimensional problem here proposed.

## 2 General Solution Using the Integral Transform Technique

The simultaneous heat, mass and pressure transfer in porous media can be expressed by a set of three parabolic equations, defined in the finite region  $V$  with boundary surface  $S$ , coupled through both diffusion equations and boundary conditions (MIKHAILOV & ÖZISIK, 1984; RIBEIRO, COTTA & MIKHAILOV, 1993):

$$w_k(x) \frac{\partial \theta_k(x,t)}{\partial t} + L_k \theta_k = P_k(x,t, \theta_1, \theta_2, \theta_3), x \in V, t > 0, k = 1,2,3 \quad (1,2,3)$$

and initial and boundary conditions given respectively by:

$$\theta_k(x,0) = f_k(x), x \in V, k = 1,2,3 \quad (3,4,5)$$

$$B_k \theta_k(x,t) = \phi_k(x,t, \theta_1, \theta_2), x \in S, k = 1,2,3 \quad (6,7,8)$$

where the equation and boundary operators are written as:

$$L_k \equiv -\nabla \cdot K_k(x) \nabla + d_k(x), k = 1,2,3 \quad (9,10,11)$$

$$B_k \equiv [\gamma_k(x) + \delta_k K_k(x) \frac{\partial}{\partial \eta}], k = 1,2,3 \quad (12,13,14)$$

$\gamma_k(x)$  and  $\delta_k(x)$  are prescribed boundary condition coefficients.  $\eta$  is the outward drawn normal to surface  $S$ . The non-linear source terms,  $P_k$  and  $\phi_k$ , may incorporate any coupling between the two potentials.

After, the approach deals with the selection of filtering solutions to minimize the non-homogeneities effects:

$$\theta_k(x,t) = \theta_{ks}(x,t) + \theta_{kh}(x,t) \quad (15)$$

where the quasi-steady solutions  $\theta_{ks}$ 's, include the consideration of characteristic representations of the equation and boundary source terms.

The resulting formulation for the filtered potential  $\theta_{kh}(x,t)$ , is obtained from the solution of the following easier problems:

$$L_k \theta_{ks} = P_{ks}(x,t), x \in V, k = 1,2,3 \quad (16,17,18)$$

$$B_k \theta_{ks} = \phi_{ks}(x,t), x \in S, k = 1,2,3 \quad (19,20,21)$$

Following the formalisms, the homogeneous system (RIBEIRO, COTTA & MIKHAILOV, 1993; COTTA & MIKHAILOV, 1997; DUARTE & RIBEIRO, 1997) could be solved exactly through the integral transform technique, since the solution of the following independent auxiliary decoupled eigenvalue problems ( $k=1,2$ ) of Sturm-Liouville type is available,

$$L_1 \Psi_i(x) = \mu_i^2 w_1(x) \Psi_i(x), x \in V, i = 1,2... \quad (22)$$

$$L_2 \Gamma_i(x) = \lambda_i^2 w_2(x) \Gamma_i(x), x \in V, i = 1,2... \quad (23)$$

with boundary conditions:

$$B_1 \Psi_i(x) = 0, x \in S, i = 1,2... \quad (24)$$

$$B_2 \Gamma_i(x) = 0, x \in S, i = 1,2... \quad (25)$$

Through the associated orthogonality property of the eigenfunctions, we define the integral transform pair as follows:  
Transform:

$$\bar{\theta}_{1,i}(t) = \int_V w_1(x) \frac{\Psi_i(x)}{N_i^{1/2}} \theta_{1h}(x,t) dv \quad (26)$$

Inverse:

$$\theta_{1h}(x,t) = \sum_{i=1}^{\infty} \frac{\Psi_i(x)}{N_i^{1/2}} \bar{\theta}_{1i}(t) \quad (27)$$

and,

Transform:

$$\bar{\theta}_{2,i}(t) = \int_V w_2(x) \frac{\Gamma_i(x)}{M_i^{1/2}} \theta_{2h}(x,t) dv \quad (28)$$

Inverse:

$$\theta_{2h}(x,t) = \sum_{i=1}^{\infty} \frac{\Gamma_i(x)}{M_i^{1/2}} \bar{\theta}_{2i}(t) \quad (29)$$

The normalizations integrals are:

$$N_i = \int_V w_1(x) \Psi_i^2(x) dv \quad (30)$$

$$M_i = \int_V w_2(x) \Gamma_i^2(x) dv \quad (31)$$

Using the integral transform methodology for the homogeneous system, after truncation to a sufficient order  $N$ , for the desired convergence, we obtain a transform constant coefficient ordinary differential equations system:

$$\frac{dY(t)}{dt} + A_{2N,2N} Y(t) = 0_{2N,1} \quad (32)$$

where

$$Y(t) = \{\bar{\theta}_{11}(\tau), \bar{\theta}_{12}(\tau), \dots, \bar{\theta}_{21}(\tau), \bar{\theta}_{22}(\tau), \dots, \bar{\theta}_{2N}(\tau)\}^T \quad (33)$$

The initial transform conditions are obtained from the filtered system. Equation (32) can be readily solved through matrix eigenvalue analysis or through well-established algorithms. Temperature and moisture potentials are computed from the explicit analytic inverse formulae on eqns (27 and 29).

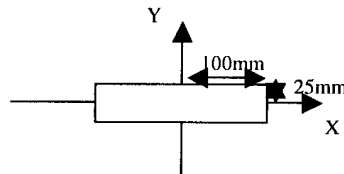


Figure 1 . Evolution of temperature profiles during the process.

### 3 Application

We consider the heat, mass and pressure balance equations written in dimensionless form, for a symmetric plate geometry as depicted in FERGUSON, LEWIS & TOMOSY (1993), Fig. (1) subjected to uniform prescribed boundary temperatures, moisture and pressure contents, and evaluated from uniform initial distributions (DUARTE, 1995, 1998). The transport coefficients are assumed constant and the problem formulation according to Luikov's theory (LUIKOV, 1975) is given by FERGUSON, LEWIS & TOMOSY (1993):

$$\frac{\partial \Theta_1(X, Y, \tau)}{\partial \tau} = K_{11} \nabla^2 \Theta_1(X, Y, \tau) + K_{12} \nabla^2 \Theta_2(X, Y, \tau) + K_{13} \nabla^2 \Theta_3(X, Y, \tau);$$

$$0 < X < 1, 0 < Y < 1; \tau > 0 \quad (34)$$

$$\frac{\partial \Theta_2(X, Y, \tau)}{\partial \tau} = K_{21} \nabla^2 \Theta_1(X, Y, \tau) + K_{22} \nabla^2 \Theta_2(X, Y, \tau) + K_{23} \nabla^2 \Theta_3(X, Y, \tau);$$

$$0 < X < 1, 0 < Y < 1; \tau > 0 \quad (35)$$

$$\frac{\partial \Theta_3(X, Y, \tau)}{\partial \tau} = K_{31} \nabla^2 \Theta_1(X, Y, \tau) + K_{32} \nabla^2 \Theta_2(X, Y, \tau) + K_{33} \nabla^2 \Theta_3(X, Y, \tau);$$

$$0 < X < 1, 0 < Y < 1; \tau > 0 \quad (36)$$

with initial conditions

$$\Theta_1(X, Y, 0) = \Theta_2(X, Y, 0) = \Theta_3(X, Y, 0) = 0; \quad 0 < X < 1, 0 < Y < 1 \quad (37,38,39)$$

and boundary conditions

$$\frac{\partial \Theta_1(0, Y, \tau)}{\partial X} = 0; \quad \frac{\partial \Theta_1(X, 0, \tau)}{\partial Y} = 0; \quad \tau > 0 \quad (40,41)$$

$$\frac{\partial \Theta_2(0, Y, \tau)}{\partial X} = 0; \quad \frac{\partial \Theta_2(X, 0, \tau)}{\partial Y} = 0; \quad \tau > 0 \quad (42)$$

$$\frac{\partial \Theta_3(0, Y, \tau)}{\partial X} = 0; \quad \frac{\partial \Theta_3(X, 0, \tau)}{\partial Y} = 0; \quad \tau > 0 \quad (43)$$

$$\Theta_1(1, Y, \tau) = \Theta_2(1, Y, \tau) = \Theta_3(1, Y, \tau) = 1; \quad \tau > 0 \quad (44,45,46)$$

$$\Theta_1(X, 1, \tau) = \Theta_2(X, 1, \tau) = \Theta_3(X, 1, \tau) = 1; \quad \tau > 0 \quad (47,48,49)$$

where the K's represent:

$$K_{11} = k_q + \varepsilon \lambda \delta'; \quad K_{12} = \varepsilon \lambda k_m; \quad K_{13} = \varepsilon \lambda k_p \quad (50,51,52)$$

$$K_{21} = \delta k_m; \quad K_{22} = k_m; \quad K_{23} = k_p \quad (53,54,55)$$

$$K_{31} = -\varepsilon \lambda k_m; \quad K_{32} = -\varepsilon k_m; \quad K_{33} = k_p (1 - \varepsilon) \quad (56,57,58)$$

and  $\theta_1$  is the dimensionless temperature distribution,  $\theta_2$  is the dimensionless moisture content distribution,  $\theta_3$  is the dimensionless pressure distribution.

Without loss of generality, using the formalisms of the *integral transform* (RIBEIRO, COTTA & MIKHAILOV 1993; COTTA & MIKHAILOV, 1997) method the solution for the system of eqs. (34,35,36) is now proposed in terms of auxiliary

problems, expressed by three pairs of easily available decoupled eigenfunction expansions of Sturm-Liouville problems, for the temperature, moisture and pressure potentials ( $k = 1,2,3$ ):

$$\frac{d^2\Psi_{ki}(X)}{dX^2} + \mu_{ki}^2\Psi_{ki}(X) = 0, \quad X \in V \quad (59)$$

$$\frac{d\Psi_{ki}(0)}{dX} = 0, \quad \Psi_{ki}(1) = 0 \quad X \in S \quad (60,61)$$

$$\frac{d^2\Gamma_{kj}(Y)}{dY^2} + \lambda_{kj}^2\Gamma_{kj}(Y) = 0, \quad Y \in V \quad (62)$$

$$\frac{d\Gamma_{kj}(0)}{dY} = 0, \quad \Gamma_{kj}(1) = 0 \quad Y \in S \quad (63,64)$$

These auxiliary problems permit the definition of the integral transform pairs that are necessary for the solution of the homogeneous problem:

Inverse,

$$\Theta_{kh}(X, Y, \tau) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{1}{N_{ki}^{1/2} M_{kj}^{1/2}} \Psi_{ki}(X) \Gamma_{kj}(Y) \bar{\Theta}_{kij}(\tau) \quad (65)$$

Transform,

$$\bar{\Theta}_{kij}(\tau) = \int_0^1 \int_0^1 \frac{\Psi_{ki}(X) \Gamma_{kj}(Y)}{N_{ki}^{1/2} M_{kj}^{1/2}} \Theta_{kh}(X, Y, \tau) dXdY \quad (66)$$

The normalizations integrals are,

$$N_{ki} = \int_0^1 \Psi_{ki}^2(X) dX \quad (67)$$

$$M_{kj} = \int_0^1 \Gamma_{kj}^2(Y) dY \quad (68)$$

The problem now is to find numerically the eigenvalues ( $\mu_{ki}$  and  $\lambda_{kj}$ ), eigenfunctions ( $\Psi_{ki}$  and  $\Gamma_{kj}$ ) and norms ( $N_{ki}$  and  $M_{kj}$ ).

The next step is to find the ordinary differential equation transform system. Using the *transform concept* in eqs. (34-58) and the auxiliary problems (59-64,67,68), after truncation to a sufficient order ( $i = 1 \dots I$ , and  $j = 1 \dots J$ ) for the desired convergence, we obtain,

$$\frac{dY(\tau)}{d\tau} + A_{2N,2N} Y(\tau) = 0_{2N,1} \quad (43)$$

where,

$$Y(\tau) = \{\bar{\Theta}_{11}(\tau) \dots \bar{\Theta}_{1N}(\tau) \quad \bar{\Theta}_{21}(\tau) \dots \bar{\Theta}_{2N}(\tau)\}^T \quad (44)$$

The initial transform conditions are similarly obtained applying the integral transform concept to the initial conditions on the homogeneous problem, resulting:

$$Y(0) = \bar{F}(\tau) \quad (45)$$

Now, this initial value problem can be solved through matrix eigenvalue analysis or scientific libraries. Initial value problem solvers with local error control schemes are employed for solving the truncated version of the transformed initial

value problem. An adaptive procedure is utilized to automatically reduce, along the integration path, the truncation orders required for a certain user-prescribed accuracy yielding, as a by-product, a global error estimator.

At this point, it is possible written the complete solution to the original problem. Using inversion formulae, temperature and moisture potentials can now be numerically obtained as:

$$\Theta_k(X, Y, \tau) = \Theta_{k_s}(X, Y) + \sum_{i=1}^I \sum_{j=1}^J \frac{\Psi_{ki}(X) \Gamma_{kj}(Y)}{M_{ki}^{1/2} N_{kj}^{1/2}} \bar{\Theta}_{kij}(\tau); \tau > 0 \quad (46)$$

where  $\Theta_{k_s}$  are the steady-state solutions,  $\Psi_{ki}$  and  $\Gamma_{kj}$  are the normalized eigenfunctions, and  $\bar{\Theta}_{kij}$  represent the transformed potentials, obtained from numerical solution of the resulting ordinary differential system, after the completion of the integral transformation process.

#### 4. Results And Discussion

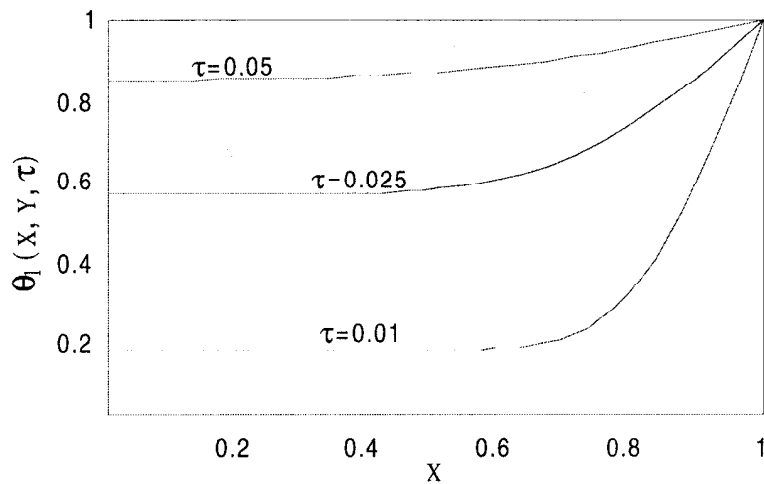
The Luikov problem as proposed above is now solved using the integral transform technique. The numerical results make it possible an inspection of the overall convergence behavior for the proposed eigenfunction expansions. The governing parameters, according to the data in LEWIS et al (1996), and CUNHA et al. (2002), assume the following values:  $\rho_0 = 1170,0 \text{ Kg.m}^{-3}$ ,  $c_q = 1.400,0 \text{ J.Kg}^{-1}.\text{°K}^{-1}$ ,  $c_m = 0,03 \text{ Kg.Kg}^{-1}.\text{°M}^{-1}$ ,  $c_p = 0,05 \text{ Kg.Kg}^{-1}.\text{Pa}$ ,  $\varepsilon = 0,3$ ,  $\lambda = 2,3.10^6 \text{ J.Kg}^{-1}$ ,  $\Delta = 0,67 \text{ °M.°K}^{-1}$ ,  $k_q = 576,0 \text{ J.h}^{-1}.\text{m}^{-1}.\text{°K}^{-1}$ ,  $k_m = 3,0.10^{-6} \text{ Kg.h}^{-1}.\text{m}^{-1}.\text{°M}^{-1}$ ,  $K_p = 1,5.10^{-6} \text{ Kg.h}^{-1}.\text{m}^{-1}.\text{Pa}^{-1}$ , and the truncation orders,  $N$ , were taken less or equal to 9, for temperature, moisture and pressure. The computer program was implemented on *Mathematica*® software (WOLFRAM, 1996), on a Pentium 700 MHz microcomputer with 256 Mb of memory RAM, and a typical run took less than 5 minutes of CPU time.

Table (1) below illustrates the convergence behavior of the two expansions (different  $N$ 's) for temperature ( $\Theta_1$ ), moisture ( $\Theta_2$ ) and pressure potential ( $\Theta_3$ ), obtained at the plate centerlines ( $Y = 0,5$ ) and different  $X$  positions. Since the heat, mass and pressure transfer processes have, for this problem, markedly different time constants, the values of dimensionless time considered in each case, are different. The convergence characteristics are, in both potentials, quite evident, with full convergence to four digits to moisture and pressure distribution and three digits to temperature distribution being achieved at  $N$  as low as 12. Such results open up broad perspectives for extension of this approach into even more involved coupled parabolic problems.

Figures (2-5) show drying process, and the temperature, moisture and pressure distributions are obtained with the converged values. As expected, the epoxy have a low thermal inertia spending about 0,1 dimensionless time to achieve the thermal equilibrium in the most deep layer, and are needed more than 1500 dimensionless times steps to the porous media to meet the moisture equilibrium. This characterize a very right mass inertia. The same can be said for the pressure, which has a very right inertia achieving the equilibrium state before the moisture content, with 1400 dimensionless time steps. The different time equilibrium make the difference in O.D.E convergence number, for moisture, because the negative pressure work over the material in the same time, carriage to deep inside a considerable quantity of humidity mass, allied to thermo-gradient effect. Such process create some difficulty to the numerical convergence, and of course, to the real drying process, as it take place after 300 dimensionless times steps, as can be seen in Fig (4). The drying process can be observed, when the pressure potential reached a half value of the prescribed boundary and the temperature is established over the material ( $\tau > 300$ ).

**Table 1 .** Convergence behavior of temperature,  $\Theta_{1T}$  moisture,  $\Theta_{2T}$  and pressure,  $\Theta_3$  expansions.

$\Theta_{1T}(X,0,0,0,01)$					$\Theta_{1T}(X,0,0,0,025)$				
X/N	3	6	9	12	X/N	3	6	9	12
0.0	0.1494	0.1790	0.1790	0.1785	0.0	0.5622	0.5636	0.5643	0.5638
0.2	0.1931	0.1781	0.1790	0.1785	0.2	0.5661	0.5642	0.5645	0.5642
0,4	0.1990	0.1783	0.1789	0.1787	0,4	0.5695	0.5680	0.5685	0.5684
0,6	0.1417	0.1845	0.1841	0.1841	0,6	0.5996	0.6012	0.6016	0.6016
0,8	0.3574	0.3215	0.3218	0.3218	0,8	0.7384	0.7359	0.7353	0.7353
1,0	1.0000	1.0000	1.0000	1.0000	1,0	1.0000	1.0000	1.0000	1.0000
$\Theta_{2T}(X,0,0,150)$					$\Theta_{2T}(X,0,0,600)$				
X/N	3	6	9	12	X/N	3	6	9	12
0.0	3.3525	3.2007	3.2030	3.2030	0.0	2.1274	2.1270	2.1270	2.1270
0.2	3.1346	3.2072	3.2057	3.2057	0.2	2.1193	2.1195	2.1195	2.1196
0,4	3.1465	3.2445	3.2440	3.2440	0,4	2.0718	2.0720	2.0720	2.0720
0,6	3.6697	3.4551	3.4575	3.4576	0,6	1.9079	1.9075	1.9075	1.9075
0,8	3.2215	3.3953	3.3928	3.3928	0,8	1.5412	1.5419	1.5419	1.5419
1,0	1.0000	1.0000	1.0000	1.0000	1,0	1.0000	1.0000	1.0000	1.0000
$\Theta_3(X,0,0,250)$					$\Theta_3(X,0,0,400)$				
X/N	3	6	9	12	X/N	3	6	9	12
0.0	0.3131	0.3220	0.3220	0.3220	0.0	0.5697	0.5709	0.5709	0.5709
0.2	0.3264	0.3216	0.3216	0.3216	0.2	0.5720	0.5713	0.5713	0.5713
0,4	0.3256	0.3213	0.3213	0.3213	0,4	0.5787	0.5782	0.5782	0.5782
0,6	0.3375	0.3480	0.3480	0.3480	0,6	0.6198	0.6211	0.6211	0.6211
0,8	0.5437	0.5349	0.5349	0.5349	0,8	0.7590	0.7579	0.7579	0.7579
1,0	1.0000	1.0000	1.0000	1.0000	1,0	1.0000	1.0000	1.0000	1.0000



**Figure 2.** Evolution of temperature profiles during the process.

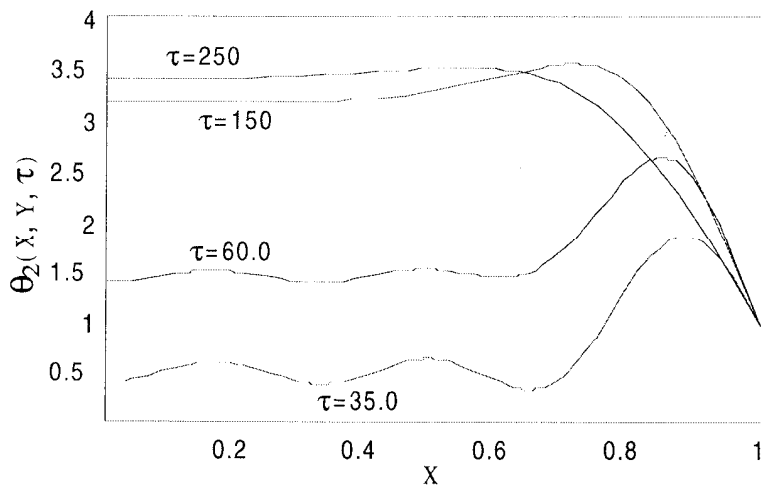


Figure 3. Evolution of moisture profiles during the process

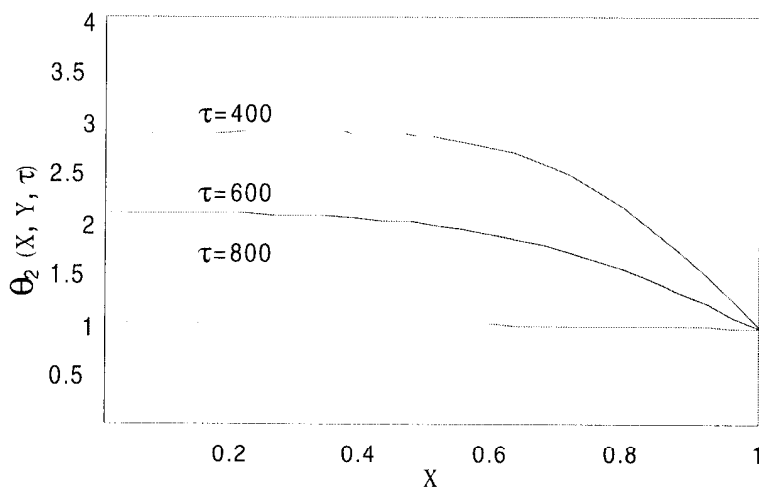


Figure 4. Evolution of moisture profiles during the process

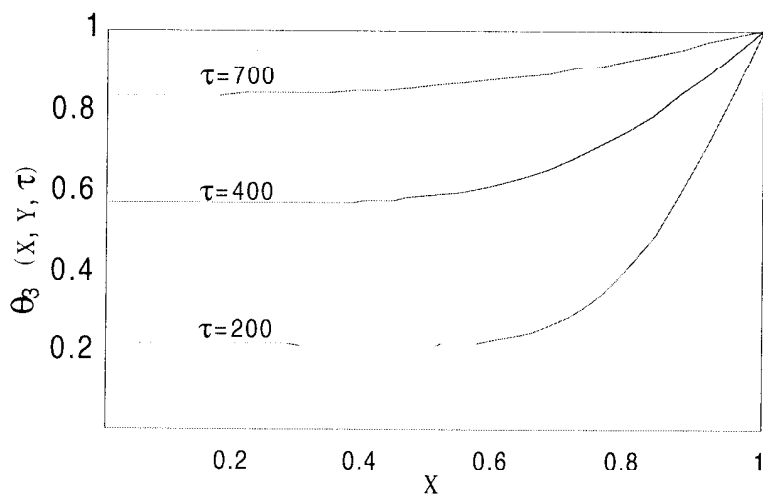


Figure 5. Evolution of pressure profiles during the process



## 5 Conclusion

In this paper the multidimensional drying problem in a capillary porous media was analytically solved for the associated temperature, moisture and pressure distributions, using Luikov's model. The generalized integral transform technique (G.I.T.T.) was applied to the problem. Convergence behavior of the adopted numerical methods and results of the temperature, moisture and pressure distributions showed very interesting aspects of drying process on such porous media.

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## 7 References

- COTTA, R.M. *Integral transforms in computational heat and fluid flow*. Boca Raton: CRC Press, 1993, 340 p.
- COTTA, R.M.; MIKHAILOV, M. D. *Heat conduction*. New York: John Wiley & Sons. 1997, 352 p.
- CUNHA, F. G. M. et al. Efeito da pressão no problema de Luikov. In: CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA. 2002, João Pessoa. *Anais...* João Pessoa: Associação Brasileira de Engenharia Mecânica, 2002, 1 CD.
- DUARTE, J.B.F. *Solução numérica da difusão de calor e massa em meios bidimensionais capilares porosos*. 1995. 96 f. Dissertação (Mestrado em Física) – Departamento de Física, Universidade Federal do Ceará, Fortaleza.
- DUARTE, J. B. F. *Abordagem analítico - numérica em meios multidimensionais capilares porosos*. 1998. 151 f. Tese (Doutorado em Física) - Departamento de Física, Universidade Federal do Ceará, Fortaleza.
- DUARTE, J. B. F.; RIBEIRO, J. W. Integral transform solution of three - dimensional Luikov problem. In: EURO THERMH 53 ADVANCED MODELLING, 1997, Mons. *Anais...* Mons: Faculte Polytechnique de Mons, 1997, p. 267 – 274.
- FERGUSON, W. J.; LEWIS, R. W.; TOMOSY, L. A finite element analysis of freeze - drying of a coffee sample. *Computer Methods in Applied Mechanics and Engineering*. North-Holand, v. 108, p 341-352, 1993.
- LEWIS, R. W. et al. *The finite element method in heat transfer analysis*. New York: John Wiley & Sons, 1996, 279 p.
- LUIKOV, A.V. System of differential equations of heat and mass transfer in capillary porous bodies (Review). *Int. Journal Heat & Mass Transfer*, London, v. 18, p. 1- 44, 1975.
- LUIKOV, A. V.; MIKHAILOV, A. Y. *Theory of energy and mass transfer*. Oxford: Pergamon Press, 1965, 324 p.
- MIKHAILOV, M. D.; ÖZISIK, M. N. *Unified analysis and solutions of heat and mass diffusion*. New York: John Wiley & Sons, 1984, 524 p.
- ÖZISIK, M. N. *Heat conduction*. New York: John Wiley & Sons, 1980, 687 p.
- RIBEIRO, J. W.; COTTA, R.M.; MIKHAILOV, M.D. Integral transform solution of Luikov's equations for heat and mass transfer in capillary porous media. *Int. Journal Heat & Mass Transfer*. London, v. 36, p. 4467-4475, 1993.
- RIBEIRO, J. W.; COTTA, R. M. On the solution of non-linear drying problems in capillary porous media through integral transformation of Luikov equations. *Int. Journal For. Num. Meth. Eng.*, New York, v. 38, p. 1001-1020, 1995.
- THOMAS, H. R.; MORGAN, K.; LEWIS, R. W. A fully non-linear analysis of heat and mass transfer problems in porous bodies. *Int. Journal For. Num. Meth. Eng.*, New York, v. 15, p.1381-1393, Feb.1980.
- WOLFRAM, S. *The mathematica book – version 3*. Cambridge: Cambridge Univ. Press, 1996, 1403 p.

## Symbols

- $a_m$  Moisture diffusion coefficient  
 $c_m$  Specific moisture capacity  
 $c_p$  Air capacity  
 $c_q$  Heat capacity

- $k_m$  Coefficient of moisture conductivity
- $k_p$  Moisture filtration coefficient
- $k_q$  Thermal conductivity
- $\theta_1$  Dimensionless temperature distribution
- $\theta_2$  Dimensionless moisture distribution,
- $\theta_3$  Dimensionless Pressure distribution,
- $X$  Dimensionless co-ordinate
- $Y$  Dimensionless co-ordinate
- $\tau$  Dimensionless time.
- $\delta$  Thermo-gradient coefficient
- $\lambda$  Latent heat
- $\epsilon$  Ratio of vapor diffusion coefficient to the coefficient of total moisture diffusion.

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