VIGA BALCÃO RETANGULAR

(*) **Lourenço Humberto Portela Reinaldo**

Prof. Titular de Resistência dos Materiais – UNIFOR

No primeiro número da Revista Tecnológica apresentamos a resolução de uma Viga Balcão Circular, para um carregamento uniforme e concentrado. Neste número, dando sequência a nosso trabalho, desenvolveremos o estudo para uma VIGA BALCÃO Retangular Fig. 1.

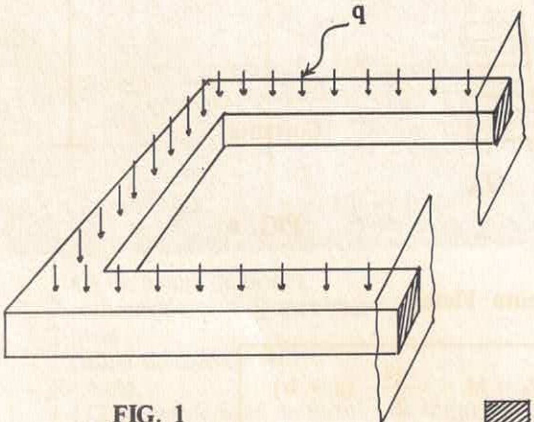
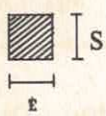


FIG. 1



Inicialmente estudaremos esta viga com carregamento uniforme.

A relação entre as rigidezas de Flexão e Torção, $K = EJ/GJ_t$, sendo $n = r/s$, e μ o coeficiente de Poisson, vale:

$$K = \frac{1+\mu}{2(1-0{,}63n)n^2}, \text{ para } r \leqslant s$$

$$\frac{1+\mu}{2(1-\frac{0{,}63}{n})}, \text{ para } r > s$$

Devido a simetria da estrutura, temos que os valores do momento fletor e esforço cortante nos engates, valem:

$$M = -q\frac{a}{2}(a+b) \quad \text{(I)}$$

$$Q = q\left(a + \frac{b}{2}\right) \quad \text{(II)}$$

Tomaremos como sistema isostático principal a estrutura abaixo, obtida liberando-se um dos engates extremos da viga. A icógnita hiperestática X_1 será apenas momento torçor em A, já M e Q são conhecido fig. 2 e 3

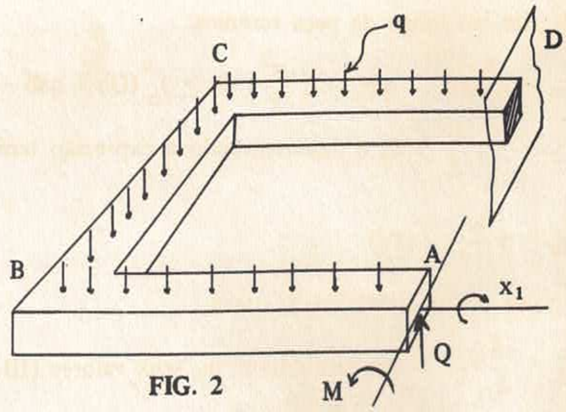


FIG. 2

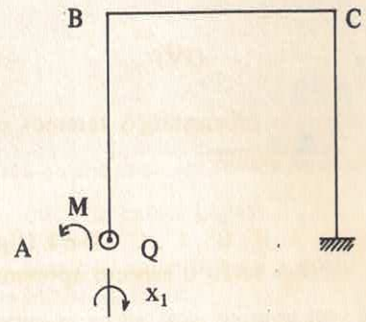


FIG. 3

para $X_1 = 1$ no isostático principal, teremos:

$$\text{Trecho AB,} \begin{cases} \bar{M}f_1 = 0 \\ \bar{M}t_1 = 1 \end{cases}$$

$$\text{BC,} \begin{cases} \bar{M}f_1 = 1 \\ \bar{M}t_1 = 0 \end{cases}$$

$$\text{CD,} \begin{cases} \bar{M}f_1 = 0 \\ \bar{M}t_1 = -1 \end{cases}$$

Usando-se o princípio dos trabalhos virtuais, teríamos: $\delta_{11} = \int \bar{M}f_1 . \bar{M}f_1\, dx + K \int \bar{M}t_1 . \bar{M}t_1\, dx$, em toda a peça, e assim sendo teremos:

$$\delta_{11} = K\int_0^a 1.1dx + \int_0^b 1.1dx + K\int_0^a (-1)(-1)dx$$

$$\delta_{11} = Ka + b + Ka \qquad \boxed{\delta_{11} = 2Ka + b} \qquad \text{(II)}$$

Devido ao carregamento, no isostático principal, teremos:

$$\text{AB} \begin{cases} Mf_0 = Qx + M - q\dfrac{x^2}{2} \\ Mt_0 = 0 \end{cases}$$

$$\text{BC} \begin{cases} Mf_0 = Qx - qax - q\dfrac{a^2}{2} \\ Mt_0 = -Qa - M + q\dfrac{a^2}{2} \end{cases}$$

$$\text{CD} \begin{cases} Mf_0 = Q(x-a) - M + qa\left(\dfrac{a}{2} - x\right) - \\ \qquad - qbx - q\dfrac{x^2}{2} \\ Mt_0 = -Qb + qab + q\dfrac{b^2}{2} \end{cases}$$

Sabendo-se que $\delta_{10} = \int \bar{M}f_1\, Mf_0\, dx + K \int \bar{M}t_1$

M_{t_0} dx, ao longo da peça teremos:

$$\delta_{10} = \int_0^b (Qx - qax - q\frac{x^2}{2})\,dx + \int_0^a (Qb - qab - q\frac{b^2}{2})\,dx$$ e desenvolvendo a expressão teremos:

$$\delta_{10} = q\frac{b^3}{12} \quad \text{(IV)}$$

O valor do hiperestático X_1, será

$X_1 = \frac{-\delta_{10}}{\delta_{11}}$ e substituindo-se os seus valores (III e IV) teremos:

$$X_1 = \frac{-qb^3}{12(2Ka + b)} \quad \text{(IV)}$$

Determinando o valor do hiperestático teremos o esforço P numa secção qualquer:

$P = P_O + X_1P_1$

Tomaremos as secções: A, B', B", C', C", D e E (fig. 4). Os esforços nestas secções terão o aspecto apresentado nas figs. 5 e 6.

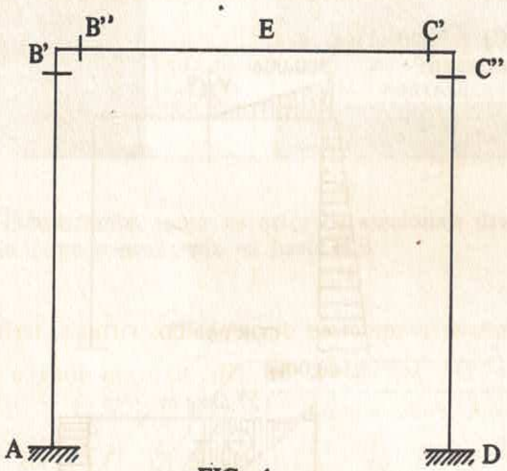


FIG. 4

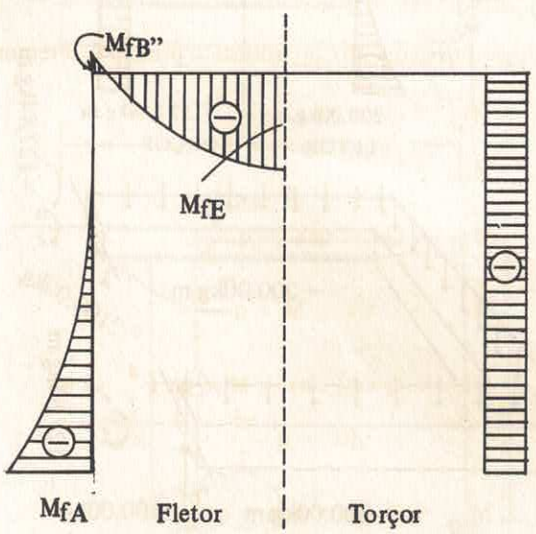


FIG. 5

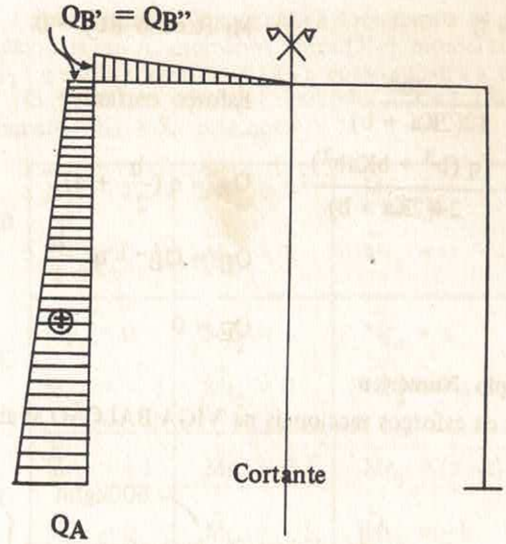


FIG. 6

Momento Fletor

$$\boxed{M_{fa} = M = \frac{-qa}{2}(a + b)}$$

$$M_{fB'} = Q^a + M - q\frac{a^2}{2}$$

$$M_{fB'} = 0$$

$$M_{fB''} = \frac{-qb^3}{12(2Ka + b)}$$

$$M_{fE} = Q\frac{b}{2} - qa.\frac{b}{2} - q\frac{b^2}{8} + \frac{-qb^3}{12(Ka+b)}$$

$$M_{fE} = \frac{q\,(b^3 + 6Kab^2)}{24(2Ka + b)}$$

Momento Torçor

$M_{tA} = X_1 = \frac{-qb^3}{12(2Ka + b)}$ constante no trecho AB

M_t (trecho BC) = 0 (não tem torção no trecho BC)

Esforço constante

$$Q_A = Q = q\,(\frac{b}{2} + a)$$

$$Q_{B'} = Q_{B''} = Q - qa \Rightarrow Q_{B'} = Q_{B''} = q.\frac{b}{2}$$

$$Q_E = 0$$

Resumindo:

Momento Fletor

$$M_{fA} = \frac{-qa(a+b)}{2}$$

Momento Torçor

$$M_t(\text{trecho AB}) = \frac{-qb^3}{12(2Ka+b)}$$

$M_{fB'} = 0$

$$M_{fB''} = \frac{-qb^3}{12(2Ka + b)}$$

$$M_{fE} = \frac{q\,(b^3 + bKab^2)}{24(2Ka + b)}$$

M_t (trecho BC) = 0

Esforço cortante

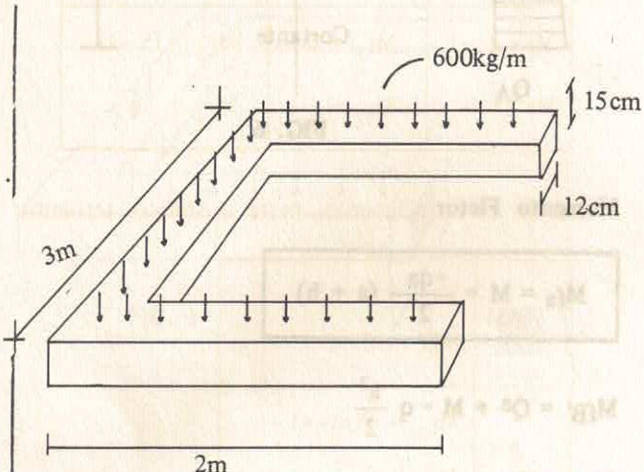
$$Q_A = q\left(\frac{b}{2} + a\right)$$

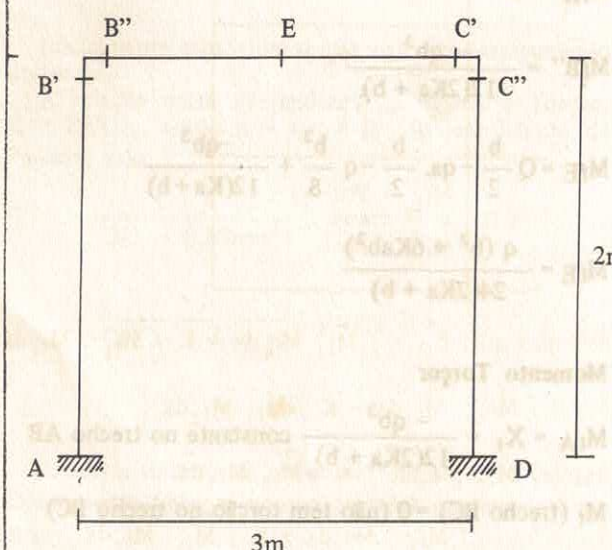
$$Q_{B'} = Q_{B''} = q.\frac{b}{2}$$

$$Q_E = 0$$

Exemplo Numérico:

Achar os esforços seccionais na VIGA BALCÃO abaixo:





Sequência de cálculo

1) Cálculo de K

$r < s$

$$N = \frac{12}{15} = 0,8$$

tomando-se para o concreto o coef. de Poisson n = 0,2 teremos:

$$M_{fB''}\,K = \frac{1 + 0,2}{2(1 - 0,63 \times 0,8).0,8^2}$$

onde $\boxed{K = 1,89}$

2) Esforços seccionais

$$M_{fA} = -600 \times \left(\frac{2}{4}\right)(2 + 3) \qquad \boxed{M_{fA} = -300\text{Kgm}}$$

$$M_{fB''} = \frac{-600 \times 3^3}{12(2 \times 1,89 \times 2 + 3)} = -127,84 \text{ Kgm}$$

$$M_{fE} = 600\,\frac{3^3 + 6 \times 1,89 \times 2 \times 3^2}{24(2 \times 1,89 \times 2 + 3)} = 547,16 \text{ Kgm}$$

Cortante

$$Q_A = 600\left(\frac{3}{2} + 2\right) \Rightarrow Q_A = 2100\text{Kg}$$

$$Q_{B'} = Q_{B''} = 600 \times \frac{3}{2} = Q_B = Q_{B''} = 900 \text{ Kg}$$

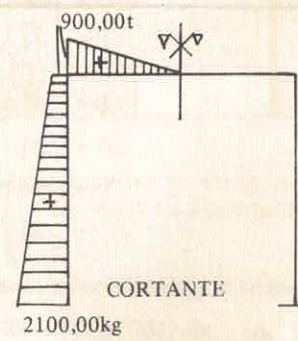
$$Q_C = 0$$

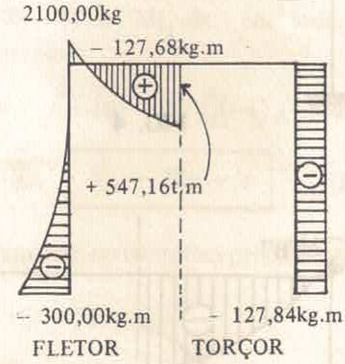
Torçor

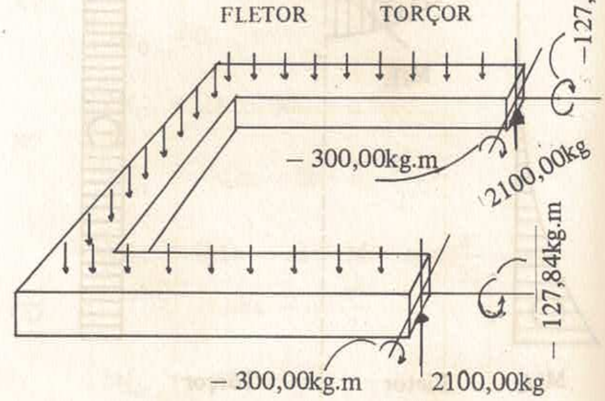
$$M_t(AB) = \text{Constante} = \frac{-600 \times 3^3}{12\,(2 \times 1,89 \times 2 + 3)}$$

$M_t(AB) = -127,84$ Kgm

$M_t(BC)$ = Nulo







Reações de Apoio

Estudaremos agora os esforços seccionais devidos a uma carga concentrada na haste AB

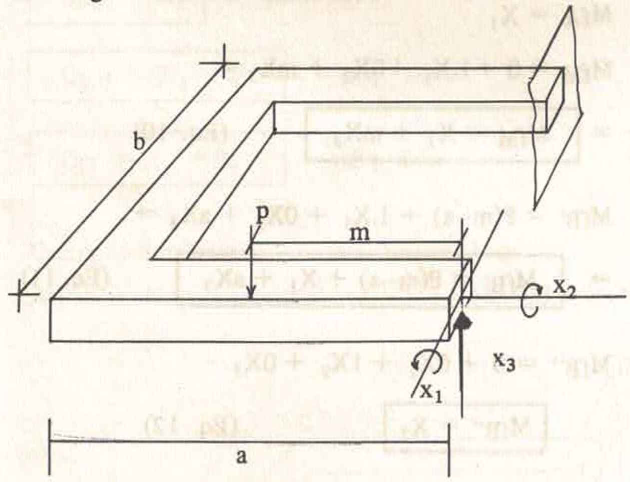


FIG. 7

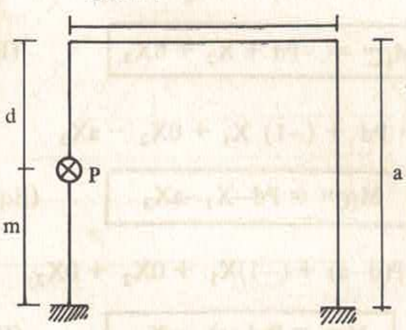


FIG. 8

Como icógnitas hiperestática tomaremos os esforços seccionais em A, momento fletor (X_1), momento torçor (X_2) e esforço constante (X_3), como mostra a figura 7.

No isostático principal, fazendo $X_1 = 1$, $X_2 = 1$ e filamente $X_3 = 1$, teríamos:

AB	$\overline{Mf}_1 = 1$ $\overline{Mt}_1 = 0$	$\overline{Mf}_2 = 0$ $\overline{Mt}_2 = 1$	$\overline{Mf}_3 = x$ $\overline{Mt}_3 = 0$
BC	$\overline{Mf}_1 = 0$ $\overline{Mt}_1 = -1$	$\overline{Mf}_2 = 1$ $\overline{Mt}_2 = 0$	$\overline{Mf}_3 = x$ $\overline{Mt}_3 = -a$
CD	$\overline{Mf}_1 = -1$ $\overline{Mt}_1 = 0$	$\overline{Mf}_2 = 0$ $\overline{Mt}_2 = -1$	$\overline{Mf}_3 = (x-a)$ $\overline{Mt}_3 = -b$

Devido o carregamento, neste isostático, teríamos:

M_O

AB	AM	$Mf_O = 0$ $Mt_O = 0$
	AB	$Mf_O = P(m-x)$ $Mt_O = 0$
	BC	$Mf_O = -Px$ $Mt_O = P.d$
	CD	$Mf_O = P(d-x)$ $Mt_O = Pb$

Sabemos que, $\delta_{11} = \int \overline{Mf}_1\ \overline{Mf}_1 dx + K \int \overline{Mt}_1 \cdot \overline{Mt}_1 dx$

$\delta_{22} = \int \overline{Mf}_2 \cdot \overline{Mf}_2 dx + K \int \overline{Mt}_2\ \overline{Mt}_2 dx$;

$\delta_{33} = \int \overline{Mf}_3\ \overline{Mf}_3 dx + K \int \overline{Mt}_3\ \overline{Mt}_3 dx$

$\delta_{21} = \delta_{12} = \int \overline{Mf}_1 \cdot \overline{Mf}_2 dx + K \int \overline{Mt}_1 \cdot \overline{Mt}_2 dx,$

$\delta_{13} = \delta_{31} = \int \overline{Mf}_1\ \overline{Mf}_3 dx + K \int \overline{Mt}_1 \cdot \overline{Mt}_3 dx$

Calculemos

$\delta_{11} = \int \overline{Mf}_1\ \overline{Mf}_1 dx + K \int \overline{Mt}_1\ \overline{Mt}_1 dx$, em toda a estrutura.

$$\delta_{11} = \int_0^a 1\,1dx + K \int_0^b 1.1dx + \int_0^a 1.dx \therefore$$

$$\boxed{\delta_{11} = 2a + Kb} \qquad \text{(Eq. 1)}$$

$\delta_{22} = \int \overline{Mf}_2 \cdot \overline{Mf}_2 dx + K \int \overline{Mt}_2 \cdot \overline{Mt}_2 dx$

$$\delta_{22} = K\int_0^a 1.1dx + \int_0^b 1.1dx + K\int_0^a 1.1dx$$

$$\boxed{\delta_{22} = 2Ka + b} \quad \text{(Eq. 2)}$$

$$\delta_{33} = \int \overline{Mf_3}.\overline{Mf_3}dx + K\int \overline{Mt_3}.\overline{Mt_3}dx$$

$$\delta_{33} = \int_0^a x.xdx + \int_0^b x.xdx + K\int_0^b a.adx + \int_0^a (x-a)$$

$$(x-a)dx + K\int_0^a b.bdx$$

$$\boxed{\delta_{33} = 2a^3/_3 + b^3/_3 + Kab(a+b)} \quad \text{(Eq. 3)}$$

$$\boxed{\delta_{12} = \delta_{21} = 0} \quad \text{(Eq. 4)}$$

$$\delta_{13} = \delta_{31} = \int \overline{Mf_1}\,\overline{Mf_3}\,dx + K\int \overline{Mt_1}\,\overline{Mt_3}\,dx$$

$$\delta_{13} = \delta_{31} = \int_0^a xdx + K\int_0^b 1.adx - \int_0^a (x-a)\,.\,1dx \because$$

$$\boxed{\delta_{13} = \delta_{31} = a^2 + Kab} \quad \text{(Eq. 5)}$$

$$\delta_{23} = \delta_{32} = \int \overline{Mf_2}\,\overline{Mf_3}\,dx + K\int \overline{Mt_2}\,\overline{M\mathfrak{t}_3}\,dx$$

$$\delta_{23} = \delta_{32} = \int_0^b 1.xdx + K\int_0^a 1.bdx \because$$

$$\boxed{\delta_{23} = \frac{b^2}{2} + Kab} \quad \text{(Eq. 6)}$$

$$\delta_{10} = \int \overline{Mf_1}.\,Mf_O\,dx + K\int \overline{Mt_1}\,Mt_O\,dx$$

$$\delta_{10} = \int_m^a 1.p(m-x)dx - K\int_0^b P.d.dx - \int_0^a 1.P(d-x)dx$$

$$\boxed{\delta_{10} = P(ma - Kdb - da - \frac{m^2}{2})} \quad \text{(Eq. 7)}$$

$$\delta_{20} = \int \overline{Mf_2}\,Mf_O\,dx + K\int \overline{Mt_2}\,Mt_O\,dx$$

$$\delta_{20} = -\int_0^b 1.P.xdx - K\int_0^a 1.P.bdx$$

$$\delta_{20} = -P(\frac{b^2}{2} + Kba) \quad \text{(Eq. 8)}$$

$$\delta_{30} = \int Mf_3\,Mf_O\,dx + K\int \overline{Mt_3}\,Mt_O\,dx$$

$$\delta_{30} = \int_m^a x.P(m-x)dx - \int_0^b x.Pxdx - K\int_0^b a.P.d.dx +$$

$$+ \int_0^a (x-a)\,P\,(d-x)\,dx - K\int_0^b b.Pbdx$$

$$\delta_{30} = P(\frac{ma^2}{2} - \frac{m^3}{6} - \frac{a^3}{6} - \frac{da^2}{2} - \frac{b^3}{3} - Kabd - Kb^2a)$$

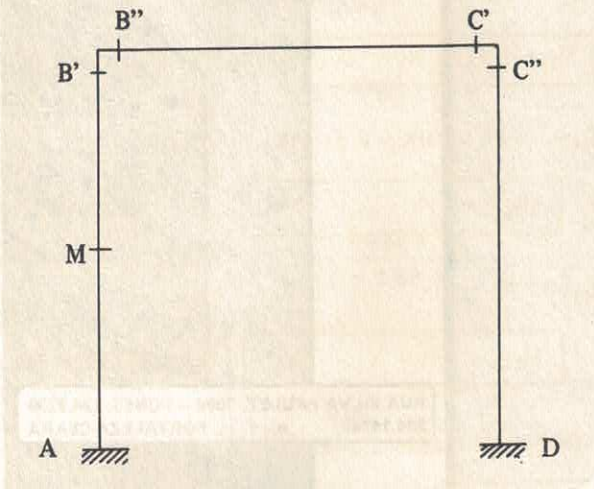
Conhecidos estes parâmetros, resolvendo o sistema de equação abaixo teremos os valores dos hiperestáticos.

$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \delta_{10} = 0$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \delta_{20} = 0$$

$$\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \delta_{30} = 0$$

Conhecidos, X_1, X_2 e X_3, teremos os esforços numa secção qualquer.
$P = P_O + P_1X_1 + P_2X_2$, sendo P um esforço qualquer: Momento ou Força.



Momento Fletor

$$Mf_A = X_1$$

$$Mf_M = 0 + 1.X_1 + 0X_2 + mX_3 =$$

$$= \boxed{Mf_M = X_1 + mX_3} \quad \text{(Eq. 10)}$$

$$Mf_{B'} = P(m-a) + 1.X_1 + 0X_2 + aX_3 \Rightarrow$$

$$\Rightarrow \boxed{Mf_{B'} = P(m-a) + X_1 + aX_3} \quad \text{(Eq. 11)}$$

$$Mf_{B''} = 0 + 0X_1 + 1X_2 + 0X_3$$

$$\boxed{Mf_{B''} = X_2} \quad \text{(Eq. 12)}$$

$$Mf_{C'} = -Pb + 0X_1 + 1X_2 + bX_3$$

$$\boxed{Mf_{C'} = -Pd + X_2 + bX_3} \quad \text{(Eq. 13)}$$

$$Mf_{C''} = Pd + (-1)\,X_1 + 0X_2 - aX_3$$

$$\boxed{Mf_{C''} = Pd - X_1 - aX_3} \quad \text{(Eq. 14)}$$

$$Mf_D = P(d-a) + (-1)X_1 + 0X_2 + 0X_3$$

$$\boxed{Mf_D = P(d-a) - X_1} \quad \text{(Eq. 15)}$$

Momento Torçor

$M_{tA} = X_2$

$M_{tB'} = 0 + 0X_1 + 1X_2 + 0X_3$

$\boxed{M_{tB'} = X_2}$ Constante na barra AB

$M_{tB''} = Pd + (-1)\, X_1 + 0X_2 - aX_3$

$\boxed{M_{tB''} = Pd - X_1 - aX_3}$ (Eq. 17)

$M_{tC'} = Pd + (-1)X_1 + 0X_2 - aX_3$

$\boxed{M_{tC'} = Pd - X_1 - aX_3}$ (Eq. 18) Const. em BC.

$M_{tC''} = Pb + 0X_1 + (-1)X_2 - bX_3$

$\boxed{M_{tC''} = Pb - X_2 - bX_3}$ (Eq. 19)

$M_{tD} = Pb + 0X_1 + (-1)\, X_2 - bX_3$

$\boxed{M_{tD} = Pb - X_2 - bX_3}$ (Eq. 20) Const. em CD

Esforço Cortante

$\boxed{Q_A = X_3}$ (Eq. 21)

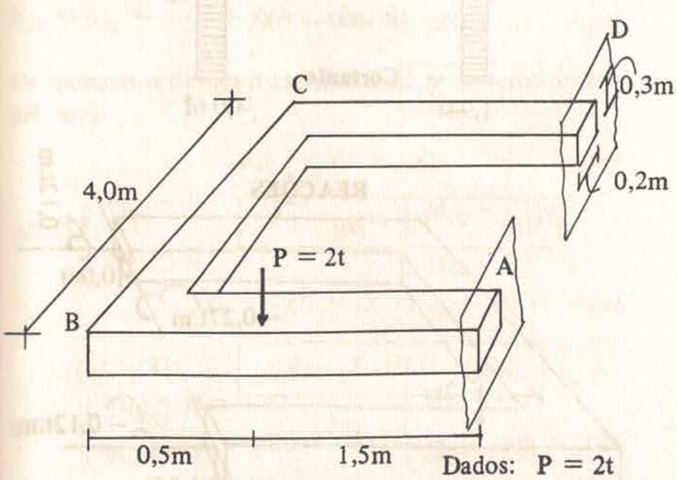
$\boxed{Q_{Me} = X_3}$ (Eq. 22)

$\boxed{Q_{Md} = X_3 - P}$ (Eq. 23)

$\boxed{Q_D = X_3 - P}$ (Eq. 24)

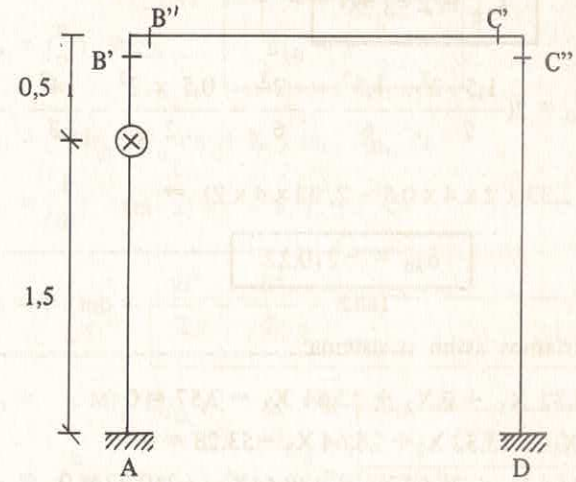
Exemplo numérico:

Achar os esforços seccionais na VIGA BALCÃO abaixo:



Dados: $P = 2t$
$a = 2m$
$b = 4m$

$m = 1{,}5m$
$d = 0{,}5m$
$\mu = 0{,}2$ (Coef. Poisson)



Sequência de Cálculo

1) Cálculo de K

$$n = \frac{0{,}2}{0{,}3} \qquad r < s$$

$$K = \frac{1 + 0{,}2}{2(1 - 0{,}63\,\frac{0{,}2}{0{,}3})\,(\frac{0{,}2}{0{,}3})^2} \qquad \boxed{K = 2{,}33}$$

2) Cálculo de $\delta_{11}, \delta_{22}, \delta_{33}, \delta_{12}, \delta_{13}, \delta_{23}, \delta_{10}, \delta_{20}, \delta_{30}$. (Equações 1,2,3,4,5,6,7,8,9)

$$\delta_{11} = 2x2 + 2{,}33 \times 4 \Rightarrow \boxed{\delta_{11} = 13{,}32}$$

$$\delta_{22} = 2 \times 2{,}33 + 4 \Rightarrow \boxed{\delta_{22} = 13{,}32}$$

$$\delta_{33} = \frac{2 \times 2^3}{3} + \frac{4^3}{3} + 2{,}33 \times 2 \times 4(2 + 4) \Rightarrow$$

$$\Rightarrow \boxed{\delta_{33} = 138{,}51}$$

$$\boxed{\delta_{12} = \delta_{21} = 0}$$

$$\delta_{31} = \delta_{13} = 2^2 + 2{,}33 \times 2 \times 4 = 22{,}64 \Rightarrow$$

$$\Rightarrow \boxed{\delta_{13} = \delta_{31} = 22{,}64}$$

$$\delta_{32} = \delta_{23} = \frac{4^2}{2} + 2{,}33 \times 2 \times 4 = 26{,}64 \Rightarrow$$

$$\boxed{\delta_{32} = \delta_{23} = 26{,}64}$$

$$\delta_{10} = 2(1{,}5 \times 2 - 2{,}33 \times 0{,}5 \times 4 - 0{,}5 \times 2 - \frac{1{,}5^2}{2}) \Rightarrow$$

$$\boxed{\delta_{10} = -7{,}57}$$

$$\delta_{20} = -2\left(\frac{4^2}{2} + 2,33 \times 4 \times 2\right) = -53,28 \Rightarrow$$

$$\Rightarrow \boxed{\delta_{20} = -53,28}$$

$$\delta_{30} = 2\left(\frac{1,5 \cdot 2^2}{2} - \frac{1,5^3}{6} - \frac{2^3}{6} - \frac{0,5 \times 2^2}{2} - \frac{4^3}{3} - \right.$$

$$\left. - 2,33 \times 2 \times 4 \times 0,5 - 2,33 \times 4 \times 2\right) \Rightarrow$$

$$\boxed{\delta_{30} = -210,22}$$

Teríamos assim o sistema:

$$13,32\,X_1 + 0\,X_2 + 22,64\,X_3 - 7,57 = 0$$

$$0\,X_1 + 13,32\,X_2 + 26,64\,X_3 - 53,28 = 0$$

$$22,64\,X_3 + 26,64\,X_2 + 138,51X_3 - 210,22 = 0$$

Resolvendo

$X_1 = -2,73\text{t}.\text{m}$

$X_2 = 0,12\text{t}.\text{m}$

$X_3 = 1,94\text{t}$

Esforços nas seções

Momento Fletor (Eq. 10, 11, 12, 13, 14, 15)

$M_{fA} = -2,73\text{t.m}$

$M_{fM} = -2,73 + 1,5 \times 1,94 \Rightarrow M_{fA} = 0,18\ \text{t}.\text{m}$

$M_{fB'} = 2\text{x}(1,5 - 2) + (-2,73) + 2 \times 1,94 \Rightarrow$

$\Rightarrow M_{fB'} = 0,15\ \text{t}.\text{m}$

$M_{fB''} = 0,12\ \text{t}.\text{m}$

$M_{fC'} = -2 \times 4 + 0,12 + 4 \times 1,94 \Rightarrow M_{fC'} = -0,12$

$M_{fC''} = 2 \times 0,5 - (-2,73) - 2 \times 1,94 \Rightarrow$

$\Rightarrow M_{fC''} = -0,15$

$M_{fD} = 2(0,5 - 2) - (-2,73) = M_{fD} = 0,27$

Momento Torçor (Eq. 16, 17, 18, 19, 20)

$M_{tA} = 0,12\ \text{t}.\text{m}$

$M_{tB'} = 0,12\ \text{t}.\text{m}$

$M_{tB''} = 2 \times 0,5 - (-2,73) - 2 \times 1,94 \Rightarrow$

$\Rightarrow M_{tB''} = -0,15\ \text{t}.\text{m}$

$M_{tC'} = 2\text{x}0,5 - (-2,73) - 2 \times 1,94 \Rightarrow$

$\Rightarrow M_{tC'} = -0,15\ \text{t}.\text{m}$

$M_{tC''} = 2 \times 4 - 0,12 - 4 \times 1,94 \Rightarrow M_{tC''} = 0,12\ \text{t}.\text{m}$

$M_{tD} = 2 \times 4 - 0,12 - 4 \times 1,94 \Rightarrow M_{tD} = 0,12\ \text{t}.\text{m}$

Esforço Cortante (Eq. 21, 22, 23, 24)

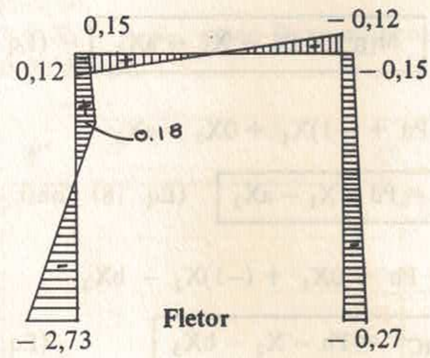
$Q_A = 1,94\text{t}$

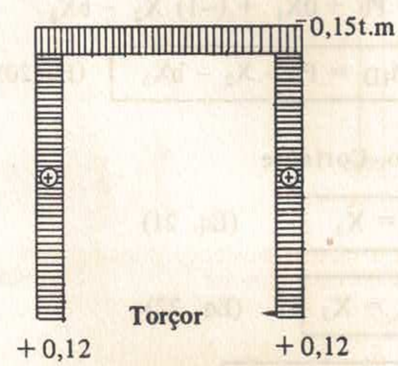
$Q_{me} = 1,94\text{t}$

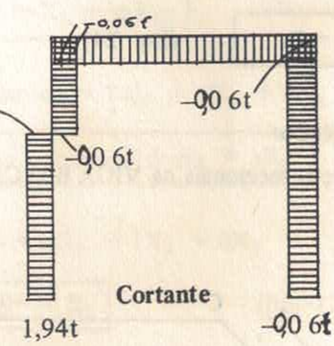
$Q_{md} = 1,94 - 2 \Rightarrow Q_{md} = -0,06\text{t}$

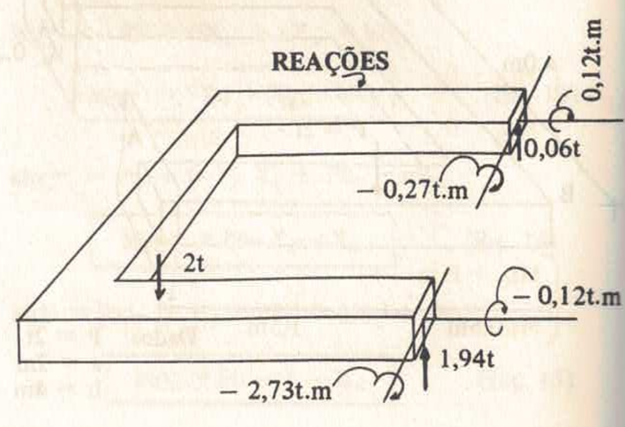
$Q_D = 1,94 - 2 = Q_D = -0,06\text{t}$

DIAGRAMAS

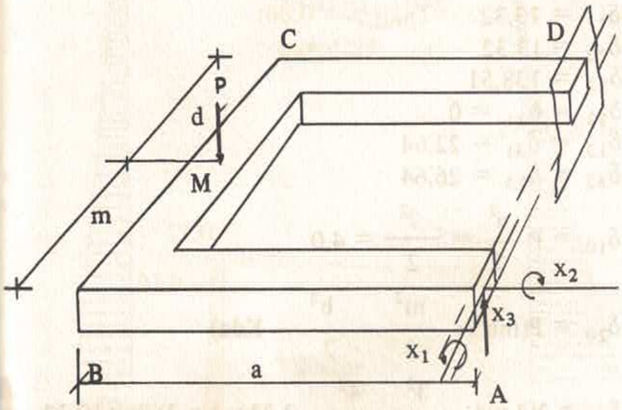


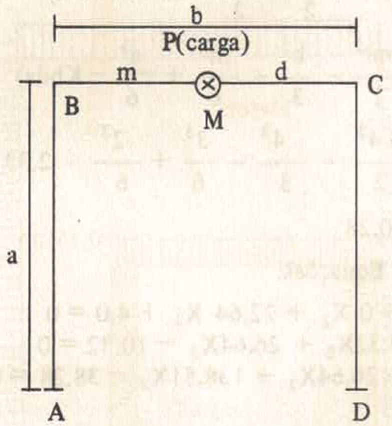






Vejamos agora com a carga localizadanno trecho BC.





Tomamos o mesmo isostético principal com as mesmas incógnitas do caso anterior.

Assim sendo

δ_{11}, δ_{22}, δ_{33}, δ_{12}, δ_{23}, δ_{13}, já são conhecidos

$\delta_{11} = 2a + Kb$ (Eq. 1)

$\delta_{22} = 2Ka + b$ (Eq. 2)

$$\delta_{33} = \frac{2a^3}{3} + \frac{b^3}{3} + Kab\,(a + b) \quad \text{(Eq. 3)}$$

$\delta_{21} = \delta_{12} = 0$ (Eq. 4)

$\delta_{13} = \delta_{31} = a^2 + Kab$ (Eq. 5)

$$\delta_{23} = \delta_{32} = \frac{b^2}{2} + Kab \quad \text{(Eq. 6)}$$

Os momentos devido o carregamento no sistema principal, será:

$$AB \begin{cases} Mto = 0 \\ Mto = 0 \end{cases} \qquad BM \begin{cases} Mto = 0 \\ Mto = 0 \end{cases}$$

$$MC \begin{cases} Mto = P(m - x) \\ Mto = 0 \end{cases} \qquad CD \begin{cases} Mto = \pm Px \\ Mto = Pd \end{cases}$$

Sabemos que:

$$\delta_{10} = \int \overline{Mf_1}, Mf_0\, dx + K \int \overline{Mt_1}\, Mt_0\, dx$$

$$\delta_{10} = \int_0^a 1.\, Pxdx \qquad \boxed{\delta_{10} = P \frac{a^2}{2}}$$

$$\delta_{20} = \int \overline{Mf_2}\, Mf_0\, dx + K \int \overline{Mt_2}\, Mt_0\, dx$$

$$\delta_{20} = \int_m^b 1.\, P(m-x)\, dx - K \int_0^a Pd\,.\, dx$$

$$\delta_{20} = P\left(mb - \frac{m^2}{2} - \frac{b^2}{2} - Kda\right)$$

$$\delta_{30} = \int \overline{Mf_3}\, Mf_0\, dx + K \int \overline{Mt_3}\, Mt_0\, dx$$

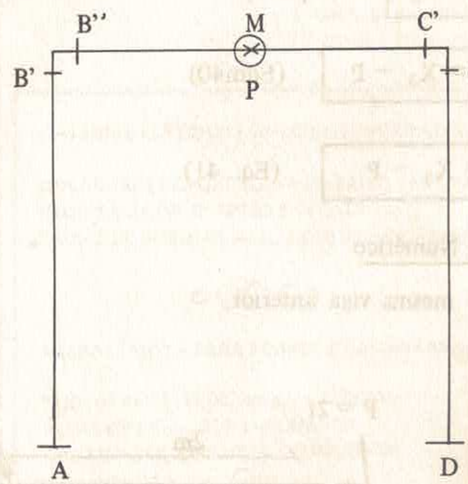
$$\delta_{30} = \int_m^b x\,.\, P(m-x)\, dx - \int_0^a P.x(x-a)\, dx - K \int_0^a b.P.d.dx$$

$$\boxed{\delta_{30} = P\left(\frac{mb^2}{2} - \frac{b^3}{3} - \frac{m^3}{6} + \frac{a^3}{6} - Kbda\right)}$$

Temos então o sistema abaixo, que nos dará os valores de X_1, X_2 e X_3.

$$\delta_{11} X_1 + \delta_{12} X_2 + \delta_{13} X_3 = 0$$
$$\delta_{21} X_1 + \delta_{22} X_2 + \delta_{23} X_3 = 0$$
$$\delta_{31} X_1 + \delta_{32} X_2 + \delta_{33} X_3 = 0$$

Com os valores de X_1, X_2 e X_3, acharemos os esforços em qualquer secção da estrutura.



Momento Fletor

$\boxed{Mf_A = X_1}$ (Eq. 25)

$\boxed{Mf_{B'} = X_1 + a\, X_3}$ (Eq. 26)

$\boxed{Mf_{B''} = X_2}$ (Eq. 27)

$\boxed{M_{fM} = X_2 + mX_3}$ (Eq. 28)

$\boxed{M_{fC'} = X_2 + b\ X_3 - Pd}$ (Eq. 29)

$\boxed{M_{fC''} = - X_1 - a\ X_3}$ (Eq. 30)

$\boxed{M_{fD} = - X_1 - Pa}$ (Eq. 31)

Momento Torçor

$\boxed{M_{tA} = X_2}$ (Eq. 32)

$\boxed{M_{tB'} = X_2}$ (Eq. 33)

$\boxed{M_{tB''} = - X_1 - aX_3}$ (Eq. 34)

$\boxed{M_{tC'} = - X_1 - a\ X_3}$ (Eq. 35)

$\boxed{M_{tC''} = - X_2 - bX_3 + Pd}$ (Eq. 36)

$\boxed{M_{tD} = - X_2 - bX_3 + Pd}$ (Eq. 37)

Esforço Cortante

$\boxed{Q_A = X_3}$)Eq. 38)

$\boxed{Q_{Me} = X_3}$ (Eq. 39)

$\boxed{Q_{Md} = X_3 - P}$ (Eqm40)

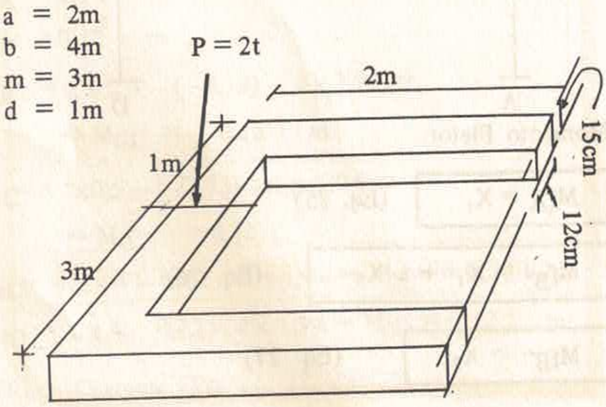
$\boxed{Q_D = X_3 - P}$ (Eq. 41)

Exemplo Numérico

Temos a mesma viga anterior

Dados:

a = 2m
b = 4m
m = 3m
d = 1m



Já conhecido:

$K = 2,33$
$\delta_{11} = 13,32$
$\delta_{22} = 13,32$
$\delta_{33} = 138,51$
$\delta_{12} = \delta_{21} = 0$
$\delta_{13} = \delta_{31} = 22,64$
$\delta_{32} = \delta_{23} = 26,64$

$$\delta_{10} = P\,\frac{a^2}{2} = \frac{2^2}{2} = 4,0$$

$$\delta_{20} = P(mb - \frac{m^2}{2} - \frac{b^2}{2} - Kda)$$

$$\delta_{20} = 2(3 \times 4 - \frac{3^2}{2} - \frac{4^2}{2} - 2,33 \times 1 \times 2) = -10,32$$

$$\delta_{30} = P\,(\frac{mb^2}{2} - \frac{b^3}{3} - \frac{m^3}{6} + \frac{a^3}{6} - Kbda)$$

$$\delta_{30} = 2(\frac{3.4^2}{2} - \frac{4^3}{3} - \frac{3^3}{6} + \frac{2^3}{6} - 2,33 \times 4 \times$$

$\delta_{30} = - 30,28$

Sistema de Equações:

$13,32\ X_1 + 0\ X_2 + 22,64\ X_3 + 4,0 = 0$
$0\ X_1 + 13,32X_2 + 26,64X_3 - 10,32 = 0$
$22,64X_1 + 26,64X_2 + 138,51X_3 - 38,28 = 0$

Resolvendo o sistema, temos:

$X_1 = - 1,19\text{t.m}; \quad X_2 = - 0,27\text{t.m}; \quad X_3 = + 0,52\text{t}$

Esforços Seccionais

Momento Fletor (Eq. 25, 26, 27, 28, 29, 30, 31)

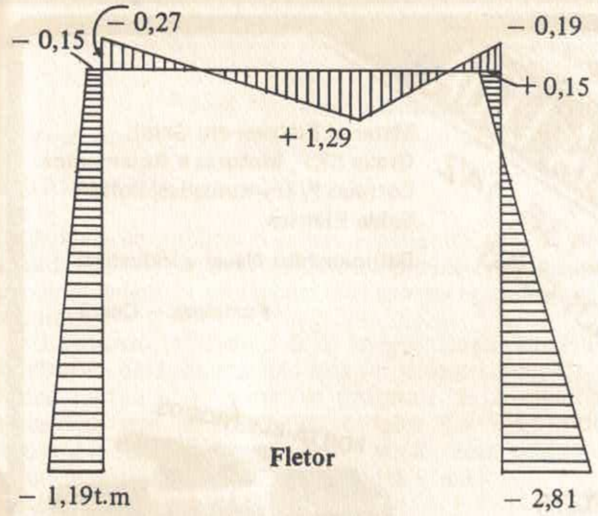
$M_{fA} = - 1,19\text{t.m}$
$M_{fB'} = 1,19 + 2 \times 0,52 = 0,15$ t.m
$M_{fB''} = - 0,27$ t.m
$M_{fM} = - 0,27 + 3 \times 0,52 = 1,29$ t.m
$M_{fC'} = - 0,27 + 4 \times 0,52 - 2 \times 1 = - 0,19$ t.m
$M_{fC''} = 1,19 - 2 \times (0,52) = 1,15$ t.m
$M_{fD} = + 1,19 - 2 \times 2 = - 2,81$

Momento Torçor (Eq. 32, 33, 34, 35,. 36, 37)

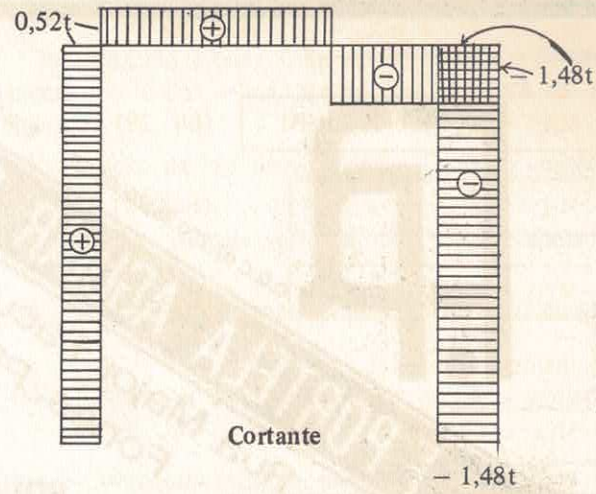
$M_{tA} = - 0,27$ t.m
$M_{tB'} = - 0,27$ t.m
$M_{tB''} = 1,19 - 2 \times 0,52 = 0,15$ t.m
$M_{tC'} = M_{tB''} = 0,15$ t.m
$M_{tC''} = 0,27 - 4 \times 0,52 + 2 \times 1 = 0,19$ t.m
$M_{tD} = M_{tC''} = 0,19$ t.m

Cortante

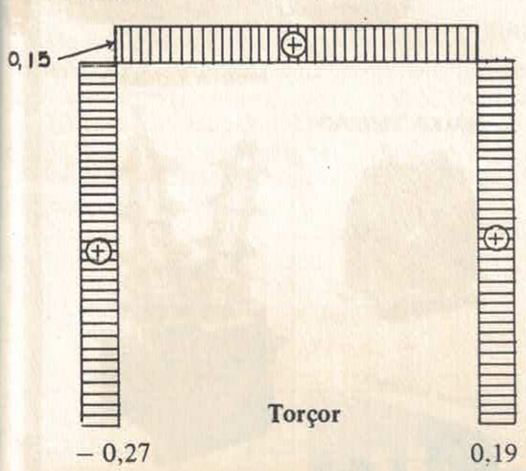
$Q_A = Q_{B'} = Q_{B''} = Q_{Me} = 0,52\text{t}$
$Q_{Md} = Q_{C'} = Q_{C''} = Q_D = 0,52 - 2 = - 1,48\text{t}$



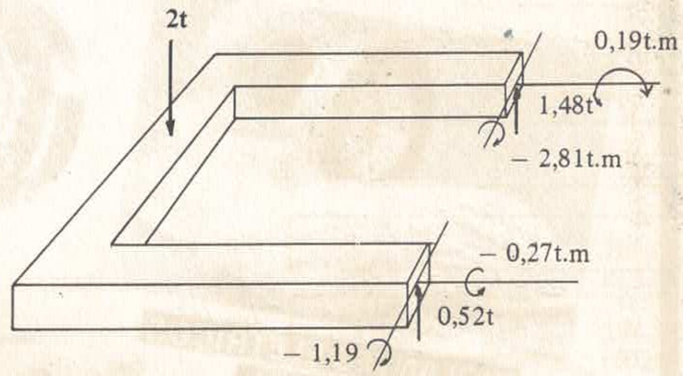
– 0,15
– 0,27
– 0,19
+ 0,15
+ 1,29
Fletor
– 1,19t.m
– 2,81



0,52t
1,48t
Cortante
– 1,48t



0,15
Torçor
– 0,27
0,19



2t
0,19t.m
1,48t
– 2,81t.m
– 0,27t.m
0,52t
– 1,19