VIGA BALCÃO TRIANGULAR

LOURENÇO HUMBERTO PORTELA
REINALDO
Prof. Titular de Resistência dos Materiais
UNIFOR

Dando prosseguimento ao nosso trabalho sobre viga balcão, desenvolvemos a solução para a determinação dos esforços seccionais numa viga balcão triangular, com carregamento uniforme e concentrado.

Chamaremos de K a relação entre as rigidezas de flexão e torção, como nos números anteriores.

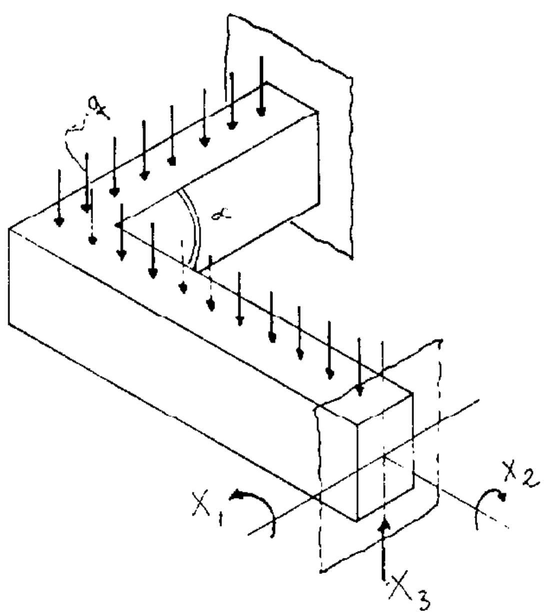
$$K = \frac{EJ}{GJt}$$

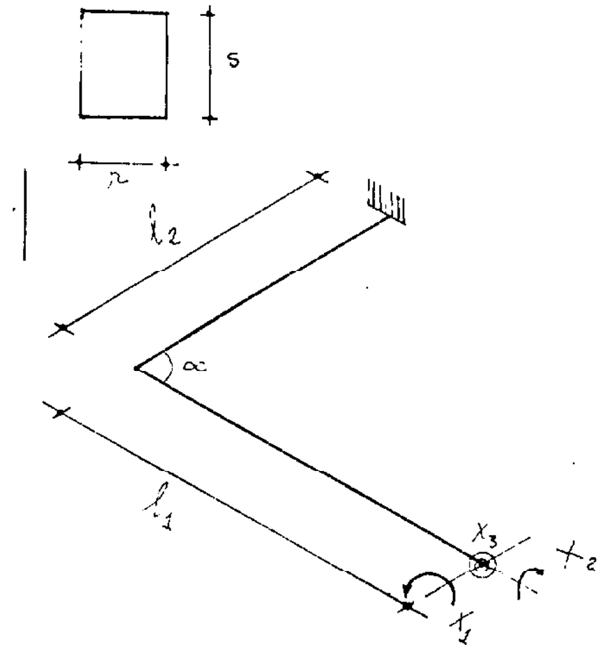
Se a seção da peça for retangular, sendo $n = \frac{r}{s}$ o coeficiente de Poisson, o valor de K será:

$$K = \frac{1+\mu}{2\,(1-0{,}63n)n2} \quad \text{para } r \leq s \text{ e}$$

$$K = \frac{1+\mu}{2\,(1-\frac{0{,}63}{n})} \quad \text{para } r > s$$

Inicialmente estudaremos a viga balcão abaixo com carregamento uniforme. (Fig. I e II)





Tomaremos como incógnitas hiperestáticas o momento fletor, momento torçor e esforço cortante na extremidade A da viga (Fig. I e II).

Na resolução deste problema usaremos o método dos esforços. Fazendo cada incógnita igual a 1 com as restantes nulas teremos:

$\overline{X}_1 = 1$

AB: $\overline{M}_{f1} = 1$, $\overline{M}_{t1} = 0$

CD: $M_{f1} = -\cos\alpha$, $M_{t1} = -\operatorname{sen}\alpha$

$\overline{X}_2 = 1$

AB: $M_{f2} = 0$, $M_{t2} = 1$

CD: $M_{f2} = \operatorname{sen}\alpha$, $M_{t2} = -\cos\alpha$

$\overline{X}_3 = 1$

AB: $M_{f3} = x$, $M_{t3} = 0$

CD: $\overline{M}_{f3} = x - \ell_1\cos\alpha$, $M_{t3} = -\ell_1 \operatorname{sen}\alpha$

Para os esforços devido ao carregamento no sistema isostático, teremos:

AB: $M_{fo} = \dfrac{-q x^2}{2}$, $M_{to} = 0$

BC: $M_{fo} = q\ell_1 \left(\dfrac{\ell_1 \cos\alpha}{2} - x\right) - \dfrac{q x^2}{2}$, $M_{to} = \dfrac{q\ell_1 2}{2} \operatorname{sen}\alpha$

Usando o princípio dos trabalhos vituáis, teremos:

$$\delta_{11} = \int M_{f1} \cdot M_{f1}\, dx + K \int M_{t1} \cdot M_{t1}\, dx$$

$$\delta_{22} = \int M_{f2} \cdot M_{f2}\, dx + K \int M_{t2} \cdot M_{t2}\, dx$$

$$\delta_{33} = \int M_{f3} \cdot M_{f3}\, dx + K \int M_{t3} \cdot M_{t3}\, dx$$

$$\delta_{1,2} = \delta_{21} = \int M_{f1} \cdot M_{f2}\, dx + K \int M_{t1} \cdot M_{t2}\, dx$$

$$\delta_{13} = \delta_{31} = \int M_{f1} \cdot M_{f3}\, dx + K \int M_{t1} \cdot M_{t3}\, dx$$

$$\delta_{32} = \delta_{23} = \int M_{f2} \cdot M_{f3}\, dx + K \int M_{t2} \cdot M_{t3}\, dx$$

$$\delta_{10} = \int m_{f1} \cdot M_{f0}\, dx - K \int M_{t1} \cdot M_{t0}\, dx$$

$$\delta_{20} = \int M_{f2} \cdot M_{f0}\, dx - K \int M_{t2} \cdot M_{t0}\, dx$$

$$\delta_{30} = \int M_{f3} \cdot M_{f0}\, dx + K \int M_{t3} \cdot M_{t0}\, dx$$

Substituindo-se os valores conhecidos nas **expressões** acima e integrando-se ao longo de toda a peça, teremos:

$$\delta_{11} = \ell_1 + \ell_2 (\cos^2 \alpha + K \operatorname{sen}^2 \alpha)$$

$$\delta_{22} = K \ell_1 + \ell_2 (\operatorname{sen}^2 \alpha + K \cos^2 \alpha)$$

$$\delta_{12} = \ell_2 (K - 1) \cos \alpha \operatorname{sen} \alpha$$

$$\delta_{13} = \frac{\ell_1^2}{2} - \ell_2 \cos \alpha \left(\frac{\ell_2}{2} - \ell_1 \cos \alpha\right) + K \ell_1 \ell_2 \operatorname{sen}^2 \alpha$$

$$\delta_{23} = \ell_2 \operatorname{sen} \left(\frac{\ell_2}{2} - \ell_1 \cos \alpha\right) + K \operatorname{sen} \alpha \cos \cdot \ell_1 \cdot \ell_2$$

$$\delta_{33} = \frac{\ell_1^3 + \ell_2^3}{3} + \ell_1 \ell_2 (\ell_1 \cos^2 \alpha - \ell_2 \cos \alpha + K \ell_1 \operatorname{sen}^2 \alpha)$$

$$\delta_{10} = q \left[-\frac{\ell_1^3}{6} + \cos \alpha \frac{\ell_2}{2} \left(\ell_1 \ell_2 - \frac{\ell_1^2}{3} \cos \alpha + \frac{\ell_2^2}{3}\right) - \frac{K \ell_1^2 \ell_2 \operatorname{sen}^2 \alpha}{2} \right]$$

$$\delta_{20} = -q \operatorname{sen} \alpha \frac{\ell_2}{2} \left[\ell_2 \ell_1 - \ell_1^2 \cos \alpha + \frac{\ell_2^2}{2} + K \cos \alpha\, \ell_1^2 \right]$$

$$\delta_{30} = q \left[-\frac{\ell_1^4}{8} - \frac{\ell_2^4}{8} - \frac{\ell_1 \ell_2^3}{3} + \frac{\ell_1 \ell_2^3}{6} \cdot \cos \alpha + \frac{3 \ell_1^2 \ell_2^2 \cos^2 \alpha}{4} - \frac{\ell_1^3 \ell_2 \cos^2 \alpha}{2} - \frac{K \ell_1^3 \ell_2 \operatorname{sen}^2 \alpha}{2} \right]$$

Com estes valores determinamos X_1, X_2 e X_3:

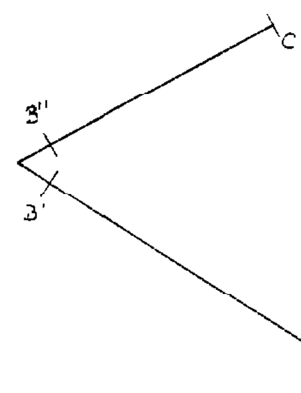
$$\delta_{11} X_1 + \delta_{12} X_2 + \delta_{13} X_3 + \delta_{10} = 0$$

$$\delta_{21} X_1 + \delta_{22} X_2 + \delta_{23} X_3 + \delta_{20} = 0$$

$$\delta_{31} X_1 + \delta_{32} X_2 + \delta_{33} X_3 + \delta_{30} = 0$$

Com os valores de X_1, X_2 e X_3 podemos achar os esforços numa seção qualquer:

$$M = M_0 + M_1 X_1 + M_2 X_2 + M_3 X_3$$



Seja encontrar os esforços seccionais na nossa viga, nas seções B', B'' e C, visto que já os conhecemos na seção A, X_1, X_2 e X_3.

$$M_{fB'} = -\frac{q\,\ell_1^2}{2} + 1X_1 + 0X_2 + \ell_1\,X_3$$

$$M_{fb''} = \frac{q\,\ell_1^2 \cos\,\alpha}{2} - \cos\alpha\,X_1 + \text{sen}\,\alpha\,X_2 -$$

$$- \ell_1 \cos\alpha\,X_3$$

$$M_{fc} = -q_1\,\ell_1\,(\ell_2 - \frac{\ell_1}{2}\cos\alpha) - \frac{q\,\ell_2^2}{2} - \cos\alpha\,X_1 +$$

$$+ \text{sen}\,\alpha\,X_2 + (\ell_2 - \ell_1\cos\alpha)\,X_3$$

Momento Torçor.

$$M_{tB'} = 0 + 0X_1 + X_2 + 0X_3$$

$$M_{tB''} = \frac{q\,\ell_1^2}{2}\,\text{sen} - \text{sen}\,\alpha\,X_1 - \cos\alpha\,X_2 -$$

$$- \ell_1\,\text{sen}\,\alpha\,X_3$$

$$M_{fc} = \frac{q\,\ell_1^2}{2}\,.\,\text{sen}\,\alpha - \text{sen}\,\alpha\,X_1 - \cos\alpha\,X_2 -$$

$$- \ell_1\,\text{sen}\,\alpha\,X_3$$

Esforço Cortante

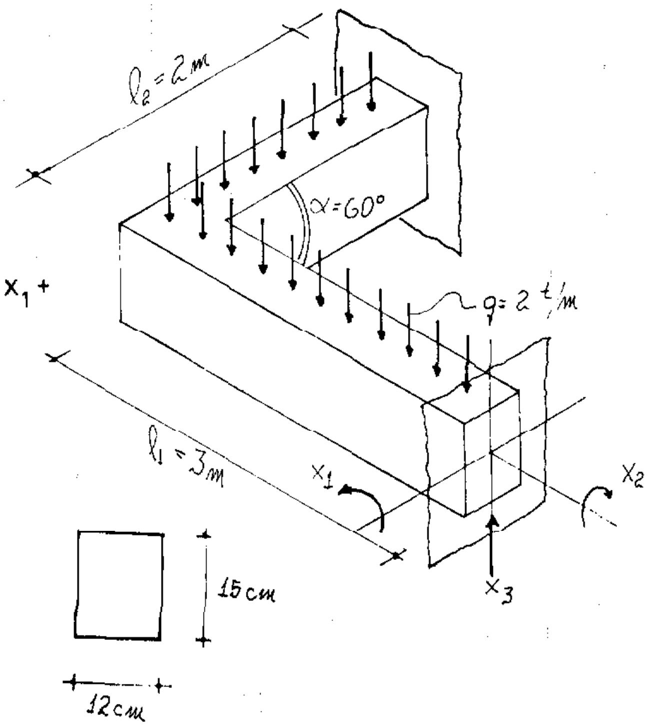
$$Q_{B'} = Q_{B''} = X_3 - q\,\ell_1$$

$$Q_c = X_3 - q\,\ell_1 - q\,\ell_2$$

Exemplo Numérico

Tomemos a viga ao lado, com o coef. de Poissoı $\mu = 0{,}2$. Neste caso teremos $r < s$, $n = \frac{12}{15} = 0{,}8$.

$$K = \frac{1 + 0{,}2}{2(1 - 0{,}63 \times 0{,}8).\,0{,}82^2} = 1{,}89$$



Com os dados do problema teremos:

$$\delta_{11} = 3 + 2\,(\cos^2 60 + 1{,}89\,\text{sen}^2 60^\circ) = 6{,}3350$$

$$\delta_{22} = 1{,}89 \times 3 + 2\,(\text{sen}^2 60 + 1{,}89\cos^2 60) = 8{,}1150$$

$$\delta_{33} = \frac{3^3 + 2^3}{3} + 3 \times 2\,(3\cos^2 60 - 2\cos 60 +$$

$$+ 1{,}89 \times 3 \times \text{sen}^2 60) = 35{,}6817$$

$$\delta_{12} = \delta_{21} = 2\,(1{,}89 - 1)\,.\,\cos 60\,\text{sen} 60 = 0{,}7708$$

$$\delta_{13} = \delta_{31} = \frac{3^2}{2} - 2\cos 60\,(\frac{2}{2} - 3\cos 60) + 1{,}89 \times$$

$$\times\, 3 \times 2\,\text{sen}^2 60 = 13{,}5050$$

$$\delta_{32} - \delta_{23} = \frac{2\,\text{sen} 60\,(2 - 3\cos 60)}{2} +$$

$$+ 1{,}89\,\text{sen} 60 \cos 60 \times 3 \times 2 = 4{,}0443$$

$$\delta_{10} = 2\left[-\frac{3^3}{6} + \cos 60 \times \frac{2}{2} \times (3\times2 - 3^2\cos 60 + \right.$$

$$\left. + \frac{2^2}{3}) - 1{,}89\,\frac{3^2 \times 2}{2}\,\text{sen}^2 60\right] = -31{,}6817$$

$$\delta_{20} = -2\,sen60 \times \frac{2}{2}\left[2\times3-3^2\cos60+\frac{2^2}{3}+1{,}89\cos60\times3^2\right] = -19{,}6386$$

$$\delta_{30} = 2\left[\frac{-3^4-2^4}{8}-\frac{2^3\times3}{3}+\frac{3\times2^3\cos60}{6}+\frac{3\times3^2\times2^2\cos60}{4}-\frac{3^3\times2\times\cos^2 60}{2}-1{,}89\times\frac{3^3\times2\ sen^2 60}{2}\right]\quad \delta_{30} = -99{,}2950$$

Teremos assim o sistema de equações abaixo:

$$6{,}3350\,X_1 + 0{,}7708\,X_2 + 13{,}5050\,X_3 - 31{,}6386 = 0$$

$$0{,}7708\,X_1 + 8{,}1150\,X_2 + 4{,}0443\,X_3 - 19{,}6386 = 0$$

$$13{,}5050\,X_1 + 4{,}0443\,X_3 + 35{,}6817\,X_3 - 99{,}2950 = 0$$

Resolvendo o sistema teremos:

$X_1 = -4{,}4516t.m$; $X_2 = 0{,}6532t.m$ e $X_3 = 4{,}3936t$

Calculemos agora os esforços nas seções A, B', B'' e C.

Momento Fletor

$$M_{fA} = X_1 = -4{,}45516t.m$$

$$M_{fB'} = -\frac{2\times3^2}{2} - 4{,}4516 + 0 + 3\times4{,}3936 = -0{,}2708t.m$$

$$M_{fB''} = \frac{2\times3^2}{2}\cos60 \times + \cos60 \times 4{,}4516 + sen60 \times 0{,}6532 - 3\times\cos60\times4{,}3936 = 0{,}7011t.m$$

$$M_{fc} = -\ 2\times3\ \left(2 - \frac{3\ \cos60}{2}\right) - \frac{2\times2^2}{2} + \cos60 \times 4{,}4516 + sen60 \times 0{,}6532 + (2-3\times\cos60)\times4{,}3936 = -6{,}5117t.m$$

Momento Torçor

$$M_{tA} = M_{tB'} = const. = X_2 = 0{,}6532t.m$$

$$M_{tB''} = M_{tC} = const. = \frac{2\times3^2}{2}\ sen60 + sen60 \times 4{,}4516 - \cos60\times0{,}6532 - 3\times sen60\times4{,}3936 = -0{,}0921t.m$$

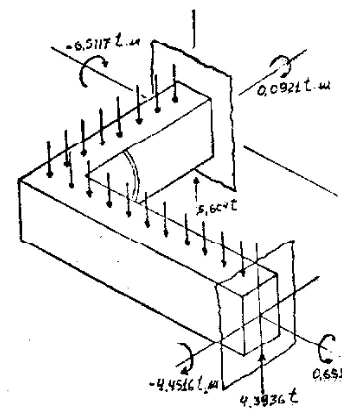
Esforçor Cortante

$$Q_A = X_3 = 4{,}3936t$$

$$Q_{B'} = Q_{B''} = 4{,}3936 - 2\times3 = -1{,}6064t$$

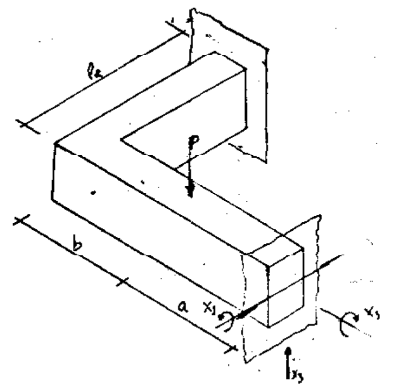
$$Q_c = -1{,}6064 - 2\times2 = -5{,}6064t$$

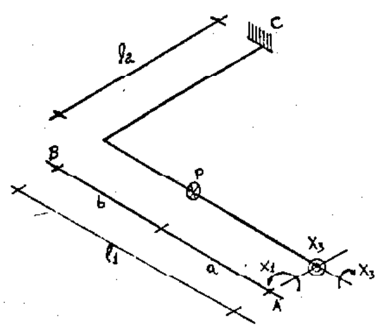
Reações de Apoio



Carga Concentrada

Tomemos o mesmo sistema anterior, liberando-se a extremidade A.





As expressões de δ_{11}, δ_{12}, δ_{13}, δ_{21}, δ_{22}, δ_{23},

δ_{31}, δ_{32} já são conhecida, restando apenas determinar δ_{10}, δ_{20} e δ_{30}.

Os momentos devido ao carregamento no sistema principal serão:

$$\text{AM} \begin{cases} M_{fo} = 0 \\ M_{to} = 0 \end{cases} \qquad \text{BM} \begin{cases} M_{fo} = P\,(a - x) \\ M_{to} = 0 \end{cases}$$

$$\text{BC} \begin{cases} M_{fo} = P\,(b \cos \alpha - x) \\ M_{to} = P\, b \,\text{sen}\, \alpha \end{cases}$$

Com estes valores determinaremos

$\delta_{10} = \int M_{f1} \cdot M_{fo}\, dx + K \int M_{t1}\, M_{to}\, dx$ e substituindo os valores de M_{t1}, M_{to} e integrando teremos:

$$\delta_{10} = P \left[a\ell_1 - \frac{a^2}{2} - \frac{\ell_1^2}{2} - b\ell_2 \cos^2 \alpha + \frac{\ell_2^2 \cos \alpha}{2} - K\, b\ell_2 \,\text{sen}^2 \alpha \right]$$

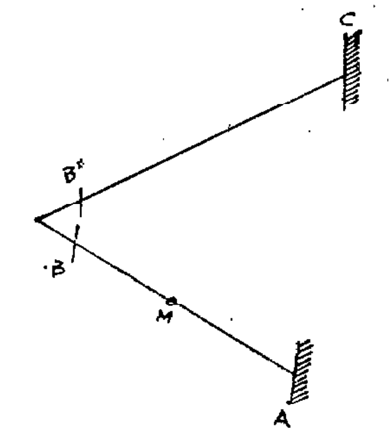
$$\delta_{20} = \int M_{f2} \cdot M_{fo}'\, dx + K \int M_{t2} \cdot M_{to}\, dx$$

$$\delta_{20} = P\ell_2 \,\text{sen}\, \alpha \left[b \cos \alpha - \frac{\ell_2}{2} - K\, b \cos \alpha \right]$$

$$\delta_{30} = \int M_{f3}\, M_{fo}\, dx + K \int M_{t3} \cdot M_{to}\, dx$$

$$\delta_{30} = P \left[\frac{a\ell_1^2}{2} - \frac{a^3}{6} - \frac{\ell_1^3}{3} - \frac{b\ell_2^2 \cos \alpha}{2} - \frac{\ell_2^3}{3} - b\,\ell_1\,\ell_2 \cos^2 \alpha + \ell_1\,\ell_2^2 \cos \alpha - K\, b\ell_1\,\ell_2 \,\text{sen}^2 \alpha \right]$$

Resolvendo da mesma forma o sistema de equações de compatibilidade de deformações, teremos os valores de X_1, X_2 e X_3 que nos possibilitaram encontrar os esforços numa seção qualquer.



Momento Fletor (Eq. I)

$$M_{fA} = X_1$$

$$M_{fM} = X_1 + aX_3$$

$$Mf_{B'} = -Pb + X_1 + \ell_1 X_3$$

$$M_f B'' = P\,b\cos\alpha - X_1 \cos\alpha + X_2 \operatorname{sen}\alpha -$$
$$- \ell_1 X_3 \cos\alpha$$

$$M_{fc} = P(\cos\alpha - \ell_2) - X_1 \cos\alpha + X2 \operatorname{sen}\alpha +$$
$$+$$
$$+ (\ell_2 - \ell_1 \cos\alpha)\, X_3$$

Momento Torçor (Eq. II)

$$M_{tA} = M_{tM} = M_{tB'} = \text{const.} = X_2$$

$$M_{tB''} = M_{tc} = \text{const.} = - X_1 \operatorname{sen}\alpha - X_2 \cos\alpha -$$
$$- \ell_1 X_3 \operatorname{sen}\alpha + Pb\operatorname{sen}\alpha$$

Esforço Cortante (Eq. III)

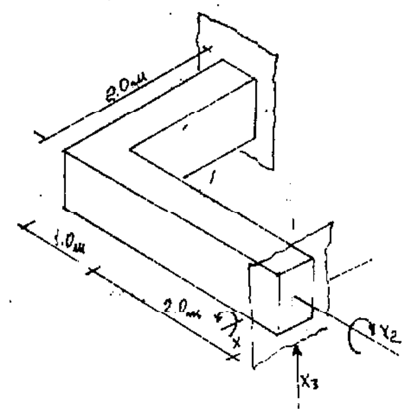
$$Q_A = Q_{Md} = X_3$$

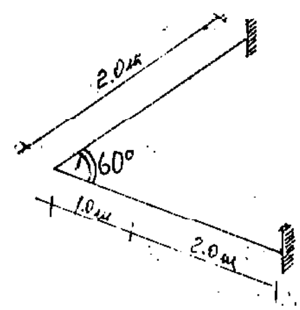
$$Q_{Me} = X_A - P$$

$$Q_{B'} = Q_{B''} = Q_c = X_A - P$$

Exemplo Numérico

Tomemos a mesma estrutura do exercício anterior, mudando apenas o carregamento.





Determinemos os valores de δ_{10}, δ_{20} e δ_{30}, pois o restante já conhecemos do exemplo anterior.

$$\delta_{10} = 2\left[2x3 - \frac{2^2 - 3^2}{2 \quad 2} - 1.2.\cos^2 60 + \right.$$
$$\left. + \frac{2^2}{2}.\cos 60 - 1{,}89 \times 1 \times 2 \times \operatorname{sen}^2 60\right] = -5{,}6700$$

$$\delta_{20} = 2x2\operatorname{sen}60\left[1x\cos 60 - \frac{2}{2} - 1{,}89 \times 1 \times \right.$$
$$\left. \cos 60\right] = -5{,}0056$$

$$\delta_{30} = 2\left[\frac{2x3^2}{2} - \frac{2^3}{6} - \frac{3^3}{3} - \frac{1x2^3}{2} \cos 60 - \right.$$
$$- \frac{2^3}{3} - 1x2x2\cos^2 60 + \frac{3x2^2}{2}\cos 60 - 1{,}89 \times 1 \times$$
$$\left. \times 3 \times 2\operatorname{sen}^2 60\right] \therefore \delta_{30} = -24{,}0100$$

Resolvendo o Sistema,

$$6{,}3350\,X_1 + 0{,}7708\,X_2 + 13{,}5050\,X_3 - 5{,}6700 = 0$$

$$0{,}7708\,X_1 + 8{,}1150\,X_2 + 4{,}0443\,X_3 - 5{,}0056 = 0$$

$$13{,}5050\,X_1 + 4{,}044\,X_2 + 35{,}681\,X_3 - 24{,}0100 = 0$$

teremos

$X_1 = -\,2{,}7791$ t.m; $X_2 = 0{,}0225$ t.m e
$X_3 = 1{,}7222$ t

Achemos os esforços nas seções A, M, B', B'' e C, usando-se as equações I, II e I I.

Momento Fletor

$$M_{fA} = -\,2{,}7791 \text{ t.m}$$

$$M_{fM} = -\,2{,}7791 + 2\times1{,}7222 = 0{,}6657 \text{ t.m}$$

$$M_{fB'} = -\,2\times1 - 2{,}7791 + 3\times1{,}7222 = 0{,}3875 \text{ t.m}$$

$$M_{fB''} = 2\times1\cos60 + 2{,}7791\cos60 + 0{,}225\,\text{sen}60 - 3\times1{,}7222\cos60 = -\,0{,}1743 \text{ t.m}$$

$$M_{fc} = 2\,(1\cos60-2) + 2{,}7791\cos60 + 0{,}0225\,\text{sen}60 + (2-3\cos60)\,1{,}7222 = -\,0{,}5545 \text{ t.m}$$

Momento Torçor

$$M_{tA} = M_{tM} = M_{tB'} = X_2 = 0.0225 \text{ t.m}$$

$$M_{tB''} = M_{tc} = 2{,}7791\,\text{sen}60 - 0{,}0225\times\cos60 - 3\times1{,}7222\,\text{sen}60 + 2\times1\times\text{sen}60 = 0{,}3468 \text{ t.m}$$

Esforço Cortante

$$Q_A = Q_{Md} = 1{,}7222\text{t}$$

$$Q_{Me} = Q_{B'} = Q_{B''} = Q_c = 1{,}7222 - 2{,}000 = -\,0{,}2778 \text{ t}$$

Reações