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# Bitcoin in Markowitz's Theory: Influence on Portfolio Efficiency

## *Bitcoin na Teoria de Markowitz: Influência na Eficiência da Carteira*

## *Bitcoin en la Teoría de Markowitz: Influencia en la Eficiencia de la Cartera*

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**Abstract**

This study investigates the impact of including Bitcoin on the efficiency of traditional investment portfolios, using Markowitz's Modern Portfolio Theory. Through an applied quantitative research approach, risk, return, correlation, and Sharpe ratio indicators were analyzed for portfolios composed of stocks from the Bovespa index, with and without Bitcoin, over five- and ten-year intervals. The results indicate that, although Bitcoin presents the highest volatility among the analyzed assets, its low correlation with traditional stocks contributes to shifting the efficient frontier to higher levels, allowing a better risk-return relationship for investors willing to tolerate greater fluctuation. From a theoretical perspective, the study expands the application of Portfolio Theory by including cryptoassets as valid diversification instruments. From a managerial perspective, it is suggested that fund managers and individual investors consider a moderate allocation to Bitcoin in order to optimize their diversification strategies and mitigate non-systematic risk, particularly in contexts of high volatility in traditional markets.

**Keywords:** Markowitz Theory, Sharpe Ratio, risk-return, portfolio optimization, Bitcoin.

***Resumo***

*Este estudo investiga o impacto da inclusão do Bitcoin na eficiência de carteiras de investimentos tradicionais, utilizando a Teoria Moderna do Portfólio de Markowitz. Por meio de uma pesquisa quantitativa aplicada, foram analisados indicadores de risco, retorno, correlação e índice de Sharpe de carteiras compostas por ações do índice Bovespa, com e sem o Bitcoin, em intervalos de cinco e 10 anos. Os resultados indicam que, embora o Bitcoin apresente a maior volatilidade entre os ativos analisados, sua baixa correlação com as ações tradicionais contribui para deslocar a fronteira eficiente para níveis superiores, permitindo melhor relação risco-retorno para investidores dispostos a tolerar maior oscilação. Teoricamente, o estudo amplia a aplicação da Teoria do Portfólio ao incluir criptoativos como instrumentos válidos de diversificação. Gerencialmente, sugere-se que gestores de fundos e investidores individuais considerem uma alocação moderada em Bitcoin para otimizar suas estratégias de diversificação e mitigação de risco não sistemático, principalmente em contextos de alta volatilidade de mercados tradicionais.*

***Palavras-chave:*** *Teoria de Markowitz, Índice de Sharpe, risco-retorno, otimização de carteira, Bitcoin.*

***Resumen***

*Este estudio investiga el impacto de la inclusión de Bitcoin en la eficiencia de carteras de inversión tradicionales, utilizando la Teoría Moderna de Portafolios de Markowitz. A través de una investigación cuantitativa aplicada, se analizaron indicadores de riesgo, rendimiento, correlación y el índice de Sharpe de carteras compuestas por acciones del índice Bovespa, con y sin Bitcoin, en intervalos de cinco y diez años. Los resultados indican que, aunque Bitcoin presenta la mayor volatilidad entre los activos analizados, su baja*

*correlación con las acciones tradicionales contribuye a desplazar la frontera eficiente hacia niveles superiores, permitiendo una mejor relación riesgo–rendimiento para los inversores dispuestos a tolerar mayores oscilaciones. Desde el punto de vista teórico, el estudio amplía la aplicación de la Teoría de Portafolios al incluir los criptoactivos como instrumentos válidos de diversificación. En términos gerenciales, se sugiere que los gestores de fondos y los inversores individuales consideren una asignación moderada en Bitcoin para optimizar sus estrategias de diversificación y mitigación del riesgo no sistemático, especialmente en contextos de alta volatilidad de los mercados tradicionales.*

***Palabras clave:*** *Teoría de Markowitz, Índice de Sharpe, riesgo–rendimiento, optimización de cartera, Bitcoin.*

## Introduction

It is estimated that approximately four out of every ten investors in financial assets hold cryptoassets (Binance, 2025). With a market capitalization of approximately US$3 trillion (Triple-A, 2024), Bitcoin stands out among more than 16,000 cryptocurrencies, accounting for roughly 63% of the total cryptocurrency market (Coingecko, 2025). Despite its global relevance, the U.S. Securities and Exchange Commission (SEC) approved Bitcoin Exchange-Traded Funds (ETFs) only in 2024 (Marins, 2025). Brazil, despite ranking among global leaders with the fifth-highest cryptocurrency adoption rate worldwide (Triple-A, 2024), remains in the public consultation phase regarding the regulation of these assets (Marins, 2025).

In this context, cryptoassets — particularly Bitcoin — have emerged as innovative financial instruments, challenging traditional financial structures and introducing new approaches to risk and return management. Introduced in 2008 by an individual or group operating under the pseudonym Satoshi Nakamoto (Nakamoto, 2008), Bitcoin was conceived as a decentralized digital currency designed to operate without intermediaries, characteristics that position it as a unique alternative during periods of economic instability (Nakamoto, 2008; Silva et al., 2020).

Due to its pronounced volatility and historically low correlation with conventional assets such as equities and government bonds, Bitcoin has attracted considerable attention from both investors and academics seeking to explore its diversification potential (Oliveira et al., 2020). Preliminary evidence suggests that incorporating Bitcoin into investment portfolios may improve allocation efficiency, particularly under conditions of macroeconomic uncertainty (Fernandes, 2023). Nevertheless, ongoing debates persist regarding how its speculative nature and associated risks affect the established principles of Modern Portfolio Theory (MPT), originally developed by Markowitz.

MPT provides a robust analytical framework for portfolio construction, relying on the relationship between expected return, risk, and asset correlations to define the efficient frontier (Markowitz, 1952, 1991). Against this backdrop, the present study aims to evaluate the influence of Bitcoin inclusion on the efficiency of portfolios constructed according to MPT principles. To achieve this objective, historical data from Brazilian assets traded on B3 and Bitcoin were analyzed over two distinct investment horizons—5 and 10 years. Portfolios were optimized according to two criteria: minimum variance and maximum Sharpe ratio. Furthermore, the results were compared with widely used market benchmarks, including the IBOV, IDIV, and ISE indices, in order to assess potential improvements in risk-return performance.

The findings demonstrate that although Bitcoin exhibits substantially higher volatility, its low correlation with other assets shifts the efficient frontier upward, enabling higher expected returns for a given level of risk. These results reinforce the role of Bitcoin as a complementary diversification instrument and broaden the practical applicability of MPT in the context of digital assets. From a managerial perspective, the study offers valuable insights for investors and fund managers by demonstrating that a carefully calibrated allocation to cryptoassets can mitigate unsystematic risk and optimize the risk-return tradeoff, particularly during periods of economic instability. By integrating empirical evidence with the classical foundations of Markowitz's theory, this work contributes to addressing gaps in the literature on alternative assets and paves the way for future research at the intersection of traditional finance and emerging digital markets.

## Literature Review

This section presents the main theoretical foundations underpinning the study. To provide a structured understanding of the phenomenon under investigation, the literature review is organized into four subsections: i) Markowitz's Modern Portfolio Theory; ii) diversification and portfolio efficiency with alternative assets; iii) Bitcoin and cryptoassets as investment assets; and iv) complementary theories, limitations of traditional models, and the Brazilian and international regulatory landscape. Together, these four dimensions constitute a fundamental basis for understanding efficient investment portfolio management (Kohatsu et al., 2023; Pakes et al., 2022), as they encompass both the technical and methodological aspects involved in optimal portfolio construction (Kohatsu et al., 2023; Neves & Silva, 2017).

The integration of these perspectives enables the development of asset allocation strategies that not only enhance portfolio resilience and diversification but also facilitate the prudent incorporation of emerging financial instruments such as cryptoassets. The literature review presented below provides a solid foundation for understanding

the relationship between risk and return, highlighting how the evolution of financial markets expands the possibilities for asset combinations while reaffirming the relevance of classical theoretical models adapted to a context of technological and financial innovation.

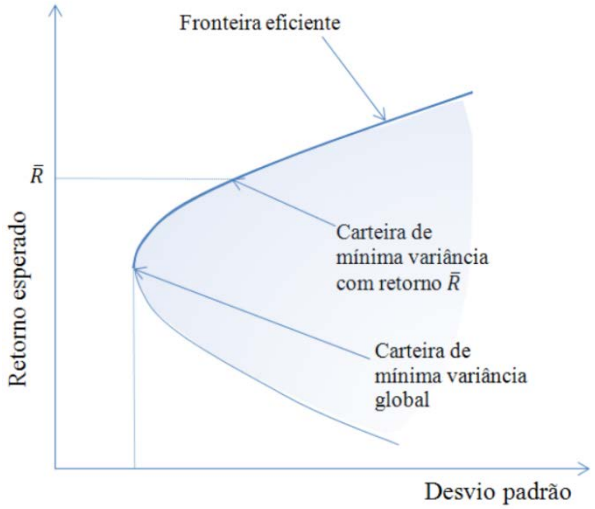
## Markowitz's Modern Portfolio Theory

MPT, developed by Harry Markowitz in 1952, revolutionized investment management by introducing a quantitative framework that considers not only the individual volatility of assets but, more importantly, the interactions among them through covariance (Markowitz, 1952, 1991). This perspective enabled investors to balance expected return and risk systematically, establishing MPT as one of the cornerstones of modern finance (Assaf, 2018; Elton et al., 2014).

At the core of the theory lies the efficient frontier, which identifies portfolios that provide the highest expected return for a given level of risk or, alternatively, the lowest risk for a desired level of return. Figure 1 illustrates this relationship, demonstrating how diversification can improve the risk-return tradeoff relative to investments concentrated in individual assets. Through variance-covariance analysis, diversification reduces unsystematic risk without sacrificing returns, thereby significantly enhancing portfolio efficiency compared with isolated investments (Kohatsu et al., 2023; Neves & Silva, 2017). This strategy is particularly valuable in volatile markets, where it contributes to greater portfolio resilience (Kohatsu et al., 2023).

**Figure 1**

Efficient frontier and the relationship between expected return and standard deviation (risk)




Source: Assaf, 2018.

Markowitz's mathematical optimization framework continues to be widely applied across a broad range of asset classes, including equities, fixed income securities, real estate investment funds, and alternative investments, demonstrating its versatility and enduring relevance (Gonçalves et al., 2024; Zanini & Figueiredo, 2005). Its fundamental principles remain essential for efficient capital allocation, adapting to new market environments while reinforcing the importance of strategic diversification in achieving an optimal balance between risk and return.

## Diversification and Portfolio Efficiency with Alternative Assets

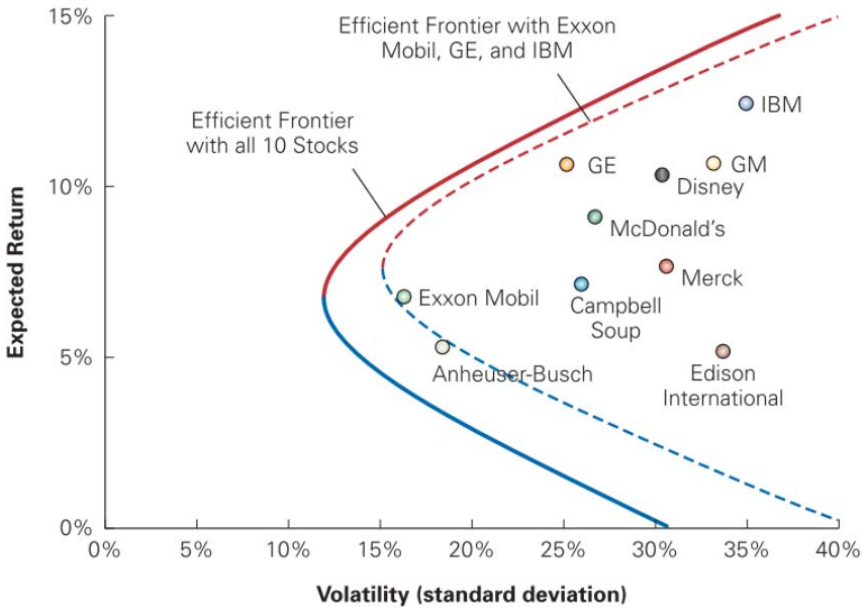
MPT established the foundations of investment management by demonstrating that diversification reduces unsystematic risk without compromising expected returns. However, the traditional literature has focused predominantly on conventional assets such as stocks and bonds, often overlooking the potential of alternative asset classes, including cryptocurrencies, to enhance portfolio optimization. While classical studies have validated the effectiveness of diversification using traditional assets (Gonçalves et al., 2024; Zanini & Figueiredo, 2005), considerably less is known about how digital assets—characterized by high volatility and low correlation with traditional markets—affect the efficient frontier.

Figure 2 illustrates classical portfolio optimization using ten stocks, showing how efficient combinations maximize returns for each level of risk. However, this approach excludes assets such as Bitcoin, which, despite their volatility, may provide unique diversification benefits due to their relative independence from conventional economic cycles (Oliveira et al., 2020). The scarcity of robust research examining cryptoassets within the MPT framework highlights a critical gap in the literature: whereas diversification benefits associated with gold or real estate are well documented, the same cannot yet be said for digital assets, whose risk and correlation patterns challenge traditional assumptions.

Recent studies indicate that incorporating alternative assets, such as real estate investment funds and gold, can stabilize portfolios during periods of uncertainty because of their lower sensitivity to macroeconomic fluctuations (Kohatsu et al., 2023; Neves & Silva, 2017). Nevertheless, selecting such assets requires careful evaluation of characteristics including liquidity, volatility, and correlations with other portfolio components. Despite their benefits, alternative assets also introduce challenges, including greater management complexity and potentially higher transaction costs. Moreover, the heterogeneity of these assets requires investors to understand their specific market dynamics, such as real estate cycles and commodity price volatility (Pakes et al., 2022).

**Figure 2**

*Efficient frontier and portfolio optimization with ten stocks.*




Source: Berk and DeMarzo, 2014.

## Bitcoin and Cryptoassets as Investment Assets

Originally conceived as a decentralized digital currency, Bitcoin has evolved into an investment asset that challenges traditional portfolio management paradigms. Studies indicate that despite its high volatility, Bitcoin has historically exhibited low correlation with equity markets, making it a powerful diversification tool within the framework of Modern Portfolio Theory (Batista & Alves, 2021; Gonçalves et al., 2024; Kohatsu et al., 2023; Zanini & Figueiredo, 2005). The cryptocurrency ecosystem has expanded considerably with the emergence of Ethereum, stablecoins, and decentralized finance (DeFi) tokens, creating new opportunities for hedging and portfolio optimization. Nevertheless, regulatory uncertainty, extreme volatility, and the relatively short historical record of these assets continue to justify moderate allocations and rigorous statistical evaluation to preserve portfolio stability.

The transition of Bitcoin from a speculative asset to a strategic diversification instrument represents a natural extension of MPT into alternative asset classes, redefining the boundaries of risk-return optimization. For academics, portfolio managers, and investors alike, the prudent incorporation of cryptoassets should not be viewed merely as a trend but rather as a logical response to the ongoing pursuit of efficiency within an increasingly dynamic and innovative financial environment.

Previous empirical studies investigating the inclusion of Bitcoin in portfolios optimized under Markowitz's framework have reported mixed, though generally positive, results regarding diversification benefits. For example, Brière et al. (2015) analyzed data from 2010 to 2013 and found that adding Bitcoin to traditional stock-and-bond portfolios improved the Sharpe ratio due to its low correlation with conventional assets, although overall portfolio volatility increased. Similarly, Platanakis and Urquhart (2019) examined portfolios between 2014 and 2018 and concluded that moderate Bitcoin allocations (up to 5%) shifted the efficient frontier upward, particularly during periods of uncertainty, while cautioning against estimation risks associated with small samples.

More recent studies, such as Mazanec (2021), using data from 2017 to 2020, confirmed efficiency gains in multi-asset portfolios, whereas Akhtaruzzaman et al. (2020) identified diversification benefits during crises such as the COVID-19 pandemic based on data from 2010 to 2019. To summarize this evidence, Table 1 presents a comparative

overview of selected studies, highlighting the authors, periods analyzed, and principal findings. Collectively, these studies suggest a tendency toward improved risk-return performance, albeit with important caveats regarding Bitcoin's substantial short-term volatility.

**Table 1**

*Empirical studies on bitcoin in Markowitz portfolios*

| Authors | Period analyzed | Main findings |
|---|---|---|
| Brière et al. (2015) | 2010-2013 | Improved Sharpe ratio; low correlation enhances diversification but increases volatility. |
| Platanakis & Urquhart (2019) | 2014-2018 | Moderate allocations shift the efficient frontier upward; risk of overestimation. |
| Mazanec (2021) | 2017-2020 | Efficiency gains in multi-asset portfolios; recommended for risk-tolerant investors. |
| Akhtaruzzaman et al. (2020) | 2010-2019 | Diversification benefits during crises; effective hedge against macroeconomic uncertainty. |

### Complementary Theories, Limitations, and the Regulatory Environment

Although MPT remains a foundational framework for portfolio optimization, its indirect reliance on the Capital Asset Pricing Model (CAPM) reveals important limitations when dealing with alternative assets such as cryptoassets. CAPM, through its single-beta approach, fails to adequately capture risks in volatile, illiquid markets that are disconnected from traditional economic factors. In this context, the Arbitrage Pricing Theory (APT), proposed by Ross (1976), emerges as a more flexible alternative by incorporating multiple macroeconomic risk factors — including inflation, interest rates, market volatility, and liquidity shocks — thereby providing a more robust framework for modeling cryptocurrency returns.

In the case of Bitcoin and other cryptoassets, the combination of high volatility, historically low correlation with conventional asset classes, and the absence of traditional valuation fundamentals renders CAPM particularly ineffective, reinforcing the appeal of APT-based approaches or extensions of the Fama-French model (1992) for more realistic and accurate asset pricing. Beyond these structural limitations, MPT assumes fully rational investors, thereby overlooking the significant behavioral biases that frequently dominate cryptocurrency markets.

From a regulatory perspective, recent developments have helped mitigate some of these risks. In the United States, the SEC's approval of spot Bitcoin ETFs in 2024 contributed to greater institutional legitimacy. In the European Union, the Markets in Crypto-Assets (MiCA) Regulation (2023) established stringent standards for transparency and anti-money laundering compliance. In Brazil, Resolution Nos. 519, 520, and 521, issued by the Central Bank of Brazil in November 2025, created the Virtual Asset Service Provider Companies (SPSAVs), regulating their authorization process and integrating cryptoasset operations into the foreign exchange and international capital markets framework (Banco Central do Brasil, 2025a).

## Method

This study was characterized as applied quantitative research. It is considered applied because it examines real assets that may be incorporated into investment portfolios. Regarding its quantitative approach, the study assessed the impact of including Bitcoin in an investment portfolio and analyzed risk and return indicators through statistical calculations. According to Silva et al. (2020), this approach is appropriate for studies that require mathematical analysis to interpret results. The selected methodology was based on Markowitz's Portfolio Theory, using metrics such as standard deviation, correlation, and the Sharpe ratio.

For mathematical modeling and statistical calculations, data were obtained from the websites b3.com.br and investing.com, which provide historical series and secondary data. Soares et al. (2022) emphasize the importance of using this type of data in quantitative analyses, as it enhances the robustness and reliability of the results.

To select the assets included in the investment portfolio under investigation, the Bovespa Index (IBOV) was used as a reference. This index comprises companies with the highest trading volume on the Brazilian stock exchange over a given period. Its composition was consulted on the B3 – Brasil, Bolsa e Balcão website (b3.com.br). A total of 11 stocks from the index were selected, each with a weighting below 4%, covering the following economic sectors: commodities, finance, energy, industry, consumer goods, and sanitation.

Although the composition of the IBOV is updated every 4 months, the assets were selected to represent a theoretical portfolio diversified across sectors, using their historical price data over the period analyzed. This approach is common in portfolio optimization studies based on MPT, in which specific differences in index composition do not compromise the validity of illustrative risk-return analyses, with emphasis placed on theoretical application rather than

exact index replication (Kohatsu et al., 2023; Zanini & Figueiredo, 2005). Thus, it was possible to select the companies used to construct the portfolio. Bitcoin (BTC) was also included in the analysis.

Asset price data were obtained from investing.com using monthly closing prices from December 2014 to December 2024, covering two economic cycles. Two main periods were considered in the analysis: the first, a 5-year horizon from 2020 to 2024, reflects the strong impact of the COVID-19 pandemic and the subsequent economic recovery; the second corresponds to the full 10-year horizon from 2014 to 2024, encompassing both the pre-pandemic and post-pandemic periods. After the monthly prices were obtained, data were organized in a Microsoft Excel spreadsheet.

Once the data were structured, monthly returns for each asset were calculated using continuously compounded returns, according to the following formula:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where represents the return in period , is the asset price in period , and s the asset price in the previous period. This return calculation method was adopted because it provides a more accurate measure of asset price variations when compared with simple returns (Assaf, 2018).

Asset volatility, or risk, was also calculated. The statistic used was the standard deviation of returns, according to the following formula:

$$\sigma = \sqrt{\frac{\sum (R_t - \bar{R})^2}{n-1}}$$

where represents the standard deviation, is the return in period , is the mean return, and is the number of periods considered. The standard deviation indicates the stability of returns over the observed period (Assaf, 2018).

Another essential indicator for the analysis was the correlation between assets, calculated for all combinations of stocks and BTC using the following formula:

$$p_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

where represents the correlation coefficient between variables *x* and *y*, is the covariance between the variables, and and are the standard deviations of *x* and *y*, respectively. This procedure enabled the construction of the correlation matrix and, subsequently, the covariance matrix.

After all components had been calculated, the Minimum-Risk Portfolio was estimated using the Excel Solver tool according to the following constraints:

- Objective: minimize portfolio risk, measured by standard deviation;
- The allocation weight of each asset must be greater than or equal to 0%;
- The sum of the allocation weights of all assets must equal 100%.

To determine the Optimal Portfolio, the Sharpe ratio (SR) was calculated. The SR measures risk-adjusted return and is defined as follows:

$$IS = \frac{R_P - R_f}{\sigma_P}$$

where SR represents the Sharpe ratio, is the expected portfolio return, is the risk-free rate, and is the portfolio standard deviation. By maximizing the SR, the best asset allocation can be identified, ensuring the most favorable risk-return tradeoff (Assaf, 2018). The maximization of the SR was also performed using the Excel Solver tool under the following constraints:

- Objective: maximize the Sharpe ratio;
- The allocation weight of each asset must be greater than or equal to 0%;
- The sum of the allocation weights of all assets must equal 100%.

Finally, it was possible to construct the efficient frontier graph and, based on all calculated information, assess the effects of including Bitcoin in the investment portfolio by comparing portfolios with and without BTC over 5- and 10-year horizons.

Monthly returns were calculated as continuously compounded returns based on raw closing prices, not adjusted for dividends or stock splits, obtained from investing.com. This choice follows established practice in studies examining

cryptoassets in Markowitz portfolios (Brière et al., 2015; Platanakis & Urquhart, 2019), since the exclusion of dividends affects the compared portfolios in a similar manner and does not alter the conclusions regarding the impact of Bitcoin on the efficient frontier.

Two main sources were used for data collection and processing. The prices of the selected stocks were obtained directly from investing.com. For each asset, the corresponding page was accessed, the "Historical Data" option was selected, the frequency was set to "Monthly," and the closing prices for the last trading day of each month were exported, covering the period from December 2014 to December 2024.

The risk-free rate, used to calculate the Sharpe ratio, was obtained from the Banco Central do Brazil (2025b) through the historical SELIC rate series. Since the Central Bank reports the SELIC rate in aggregate periods rather than as an individualized monthly series, an adjustment was performed to transform the rates into a monthly series. Each month was assigned the rate corresponding to the prevailing period, which was then converted to the same time basis as the monthly returns calculated for the assets.

All price series were obtained without missing data, as all selected assets had available quotations on the last trading day of each month throughout the entire period analyzed. After collection, the data were consolidated in a Microsoft Excel spreadsheet, enabling the calculation of monthly logarithmic returns, volatility, the correlation matrix, and the covariance matrices used in the portfolio optimization stage.

Robustness analyses were conducted to assess the stability and reliability of the main findings related to the inclusion of Bitcoin in optimized portfolios. The out-of-sample test made it possible to verify whether the observed benefits were not restricted to the estimation period, thereby reducing the risk of model overfitting. To this end, the weights estimated using data from 2014–2019 were applied to the returns observed from 2020–2024. The results indicate that the portfolio including Bitcoin maintained superior performance in terms of the Sharpe ratio when compared with the portfolio composed only of traditional assets, suggesting that the diversification gains persisted outside the original sample.

In addition, the sensitivity analysis examined how different restrictions on the maximum Bitcoin allocation affected portfolio efficiency, which is essential for evaluating the practical feasibility of including this asset under different risk profiles. The results show that, in both the five- and ten-year horizons, even with progressive reductions in the maximum Bitcoin weight, the Sharpe ratio remained consistently higher than that of the portfolio without Bitcoin. This behavior indicates that the improvement in the risk-return relationship associated with Bitcoin is robust and does not depend on excessively high allocations, remaining valid under different modeling scenarios and investment constraints.

## Results

For clarity, the results are presented in five sections: i) portfolio structure and general overview of the assets; ii) composition of the optimized portfolios without Bitcoin; iii) composition of the optimized portfolios with Bitcoin; iv) efficient frontier and portfolio simulation; and v) comparison with market indices. Each subsection is described below.

### Portfolio Structure and General Overview of the Assets

This section presents the results obtained from the simulation analysis of two investment portfolios: one composed solely of traditional assets listed on the Brazilian stock exchange (B3), and another composed of the same assets with the inclusion of Bitcoin. The indicators assessed included mean return, standard deviation (volatility), correlation, covariance, risk, return, and Sharpe ratio over 5- and 10-year time horizons, encompassing two economic cycles marked by fiscal and health crises. The objective was to identify the effects of including Bitcoin in the portfolio from the perspective of the aforementioned indicators, particularly risk and return. Based on the results, it is possible to evaluate the implications of Bitcoin-based diversification and discuss its effects on the strategies of investors and fund managers.

Before proceeding to the construction of the optimized portfolios, Table 2 presents the main descriptive statistics of the monthly logarithmic returns of the analyzed assets, as well as the results of the Jarque-Bera normality tests for the five- and ten-year horizons. Bitcoin recorded the highest average monthly return in both periods, 4.27% and 4.74%, respectively, while also exhibiting the highest volatility, 18.90% and 20.15%. Among the stocks, PRIO3 stood out for its high return, whereas ABEV3 and EQTL3 exhibited the lowest volatilities.

Regarding the distribution of returns, all assets rejected the null hypothesis of normality (p-value < 0.05), displaying skewness and/or excess kurtosis. This is a common feature of financial time series and is especially pronounced in Bitcoin and cyclical assets such as PRIO3 and RENT3. This evidence reinforces the need for caution when strictly interpreting mean-variance optimization results, although it does not preclude the practical application of Markowitz's Theory, as widely discussed in contemporary literature.

**Table 2**

*Descriptive statistics and normality tests of monthly asset returns (5- and 10-year periods)*

5-year period

| Asset | n | Mean | Standard deviation | Median | Minimum | Maximum | Skewness | Kurtosis | Jarque-Bera | p-value |
|---|---|---|---|---|---|---|---|---|---|---|
| JBSS3 | 60 | 0.96% | 8.81% | 0.77% | -19.46% | 20.73% | 0.15 | -0.29 | 27.37 | 0.01 |
| PRIO3 | 60 | 3.01% | 18.40% | 1.80% | -86.01% | 47.20% | -1.45 | 8.50 | 96.65 | 0.01 |
| BBAS3 | 60 | 0.51% | 10.15% | -0.55% | -51.44% | 16.66% | -2.18 | 10.93 | 204.69 | 0.01 |
| BBDC4 | 60 | -1.22% | 10.53% | -2.08% | -38.42% | 22.84% | -0.39 | 1.78 | 5.25 | 0.01 |
| ELET3 | 60 | 0.06% | 10.56% | 1.14% | -36.99% | 21.63% | -0.62 | 1.60 | 8.80 | 0.01 |
| EQTL3 | 60 | 0.46% | 8.32% | 0.72% | -33.68% | 21.05% | -0.87 | 3.84 | 9.35 | 0.01 |
| SBSP3 | 60 | 0.78% | 10.53% | 0.82% | -40.70% | 30.41% | -0.64 | 3.76 | 5.53 | 0.01 |
| WEGE3 | 60 | 1.97% | 10.01% | 1.31% | -25.01% | 28.77% | 0.10 | 0.48 | 15.97 | 0.01 |
| EMBR3 | 60 | 1.74% | 14.91% | 0.59% | -57.54% | 33.03% | -0.65 | 3.03 | 4.29 | 0.01 |
| ABEV3 | 60 | -0.49% | 7.32% | -1.98% | -20.52% | 18.39% | 0.08 | 1.06 | 9.43 | 0.01 |
| RENT3 | 60 | -0.48% | 12.93% | 0.25% | -63.08% | 26.25% | -1.72 | 7.87 | 89.07 | 0.01 |
| BTC | 60 | 4.27% | 18.90% | 3.41% | -46.72% | 38.50% | -0.28 | 0.06 | 22.41 | 0.01 |

10-year period

| Asset | n | Mean | Standard deviation | Median | Minimum | Maximum | Skewness | Kurtosis | Jarque-Bera | p-value |
|---|---|---|---|---|---|---|---|---|---|---|
| JBSS3 | 120 | 1.23% | 10.37% | 0.89% | -23.91% | 26.36% | -0.02 | -0.36 | 56.31 | 0.01 |
| PRIO3 | 120 | 3.72% | 19.27% | 2.89% | -86.01% | 69.31% | -0.61 | 4.35 | 16.60 | 0.01 |
| BBAS3 | 120 | 1.14% | 11.31% | -0.21% | -51.44% | 38.89% | -0.37 | 4.58 | 15.25 | 0.01 |
| BBDC4 | 120 | 0.16% | 9.90% | -0.18% | -38.42% | 23.88% | -0.21 | 1.18 | 17.53 | 0.01 |
| ELET3 | 120 | 1.61% | 14.06% | 1.71% | -36.99% | 47.42% | 0.54 | 1.32 | 19.89 | 0.01 |
| EQTL3 | 120 | 1.48% | 7.42% | 1.57% | -33.68% | 21.05% | -0.80 | 3.48 | 14.01 | 0.01 |
| SBSP3 | 120 | 1.55% | 10.35% | 2.57% | -40.70% | 31.98% | -0.51 | 2.82 | 5.34 | 0.01 |
| WEGE3 | 120 | 1.97% | 8.55% | 2.32% | -25.01% | 28.77% | 0.01 | 0.82 | 23.86 | 0.01 |
| EMBR3 | 120 | 0.72% | 12.25% | -0.19% | -57.54% | 33.03% | -0.48 | 3.95 | 9.10 | 0.01 |
| ABEV3 | 120 | 0.00% | 6.55% | -0.65% | -20.52% | 18.39% | -0.15 | 1.08 | 18.88 | 0.01 |
| RENT3 | 120 | 1.06% | 11.20% | 2.25% | -63.08% | 26.25% | -1.48 | 8.00 | 168.93 | 0.01 |
| BTC | 120 | 4.74% | 20.15% | 4.57% | -46.72% | 53.28% | -0.08 | 0.07 | 43.17 | 0.01 |

Once data had been organized, it was possible to summarize the results for each asset by evaluating average monthly return and standard deviation (volatility) over the 5- and 10-year periods, as shown in Table 3.

**Table 3**

Average monthly return and standard deviation

| Asset | Observations | Average monthly return (5 years) | Standard deviation (5 years) | Average monthly return (10 years) | Standard deviation (10 years) |
|---|---|---|---|---|---|
| JBSS3 | 121 | 0.96% | 8.81% | 1.23% | 10.37% |
| PRIO3 | 121 | 3.01% | 18.40% | 3.72% | 19.27% |
| BBAS3 | 121 | 0.51% | 10.15% | 1.14% | 11.31% |
| BBDC4 | 121 | -1.22% | 10.53% | 0.16% | 9.90% |
| ELET3 | 121 | 0.06% | 10.56% | 1.61% | 14.06% |
| EQTL3 | 121 | 0.46% | 8.32% | 1.48% | 7.42% |
| SBSP3 | 121 | 0.78% | 10.53% | 1.55% | 10.35% |
| WEGE3 | 121 | 1.97% | 10.01% | 1.97% | 8.55% |
| EMBR3 | 121 | 1.74% | 14.91% | 0.72% | 12.25% |
| ABEV3 | 121 | -0.49% | 7.32% | 0.00% | 6.55% |
| RENT3 | 121 | -0.48% | 12.93% | 1.06% | 11.20% |
| Bitcoin | 121 | 4.27% | 18.90% | 4.74% | 20.15% |

### Analysis of asset returns and volatility

Bitcoin was the asset with the highest return in both periods, but also with the greatest volatility, highlighting its highly fluctuating nature (Chen, 2023). PRIO3 also stands out for presenting volatility comparable to that of Bitcoin, although with lower returns.

When analyzing the least volatile assets, differences were observed between the periods. In the 5-year horizon, the least volatile assets were ABEV3 (7.32%), EQTL3 (8.32%), and JBSS3 (8.81%). Over the 10-year period, ABEV3 (6.55%) and EQTL3 (7.42%) maintained their positions, while WEGE3 (8.55%) replaced JBSS3 (10.37%). Several factors may be associated with this change, including economic cycles, exchange rate variation, and input costs affecting commodity-based companies. Conversely, WEGE3 consolidated its position as a solid company, offering average returns close to 2% per month.

### Analysis of asset correlation and covariance

Tables 4 and 5 present the correlation matrices among the assets, allowing the analysis of their mutual interactions. This evaluation criterion is essential within Modern Portfolio Theory, as it reveals how financial assets move relative to one another. Positive correlations indicate joint movement, whereas negative correlations indicate opposite or independent movements.

**Table 4**

*Correlation matrix of asset returns over the 5-year period*

| | JBSS3 | PRIO3 | BBAS3 | BBDC4 | ELET3 | EQTL3 | SBSP3 | WEGE3 | EMBR3 | ABEV3 | RENT3 | Bitcoin |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| JBSS3 | 1.00 | | | | | | | | | | | |
| PRIO3 | 0.34 | 1.00 | | | | | | | | | | |
| BBAS3 | 0.14 | 0.57 | 1.00 | | | | | | | | | |
| BBDC4 | 0.22 | 0.48 | 0.72 | 1.00 | | | | | | | | |
| ELET3 | 0.21 | 0.60 | 0.53 | 0.55 | 1.00 | | | | | | | |
| EQTL3 | 0.23 | 0.61 | 0.62 | 0.66 | 0.58 | 1.00 | | | | | | |
| SBSP3 | 0.19 | 0.56 | 0.49 | 0.48 | 0.56 | 0.72 | 1.00 | | | | | |
| WEGE3 | 0.09 | 0.34 | 0.13 | 0.28 | 0.35 | 0.50 | 0.41 | 1.00 | | | | |
| EMBR3 | 0.31 | 0.45 | 0.33 | 0.33 | 0.31 | 0.32 | 0.34 | 0.17 | 1.00 | | | |
| ABEV3 | 0.21 | 0.44 | 0.54 | 0.63 | 0.44 | 0.55 | 0.48 | 0.19 | 0.38 | 1.00 | | |
| RENT3 | 0.16 | 0.61 | 0.65 | 0.66 | 0.58 | 0.67 | 0.62 | 0.52 | 0.26 | 0.37 | 1.00 | |
| Bitcoin | 0.37 | 0.37 | 0.20 | 0.15 | 0.08 | 0.27 | 0.15 | 0.24 | 0.33 | 0.21 | 0.26 | 1.00 |

**Table 5**

*Correlation matrix of asset returns over the 5-year period*

| | JBSS3 | PRIO3 | BBAS3 | BBDC4 | ELET3 | EQTL3 | SBSP3 | WEGE3 | EMBR3 | ABEV3 | RENT3 | Bitcoin |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| VALE3 | 1.00 | | | | | | | | | | | |
| PETR4 | 0.12 | 1.00 | | | | | | | | | | |
| ITUB4 | 0.06 | 0.50 | 1.00 | | | | | | | | | |
| BBAS3 | 0.15 | 0.44 | 0.78 | 1.00 | | | | | | | | |
| ELET3 | 0.23 | 0.27 | 0.57 | 0.55 | 1.00 | | | | | | | |
| EQTL3 | 0.20 | 0.41 | 0.57 | 0.65 | 0.53 | 1.00 | | | | | | |
| SBSP3 | 0.12 | 0.37 | 0.46 | 0.49 | 0.49 | 0.71 | 1.00 | | | | | |
| WEGE3 | 0.05 | 0.27 | 0.15 | 0.23 | 0.19 | 0.38 | 0.31 | 1.00 | | | | |
| EMBR3 | 0.18 | 0.24 | 0.15 | 0.18 | 0.03 | 0.19 | 0.22 | 0.09 | 1.00 | | | |
| ABEV3 | 0.18 | 0.25 | 0.35 | 0.49 | 0.26 | 0.44 | 0.40 | 0.30 | 0.23 | 1.00 | | |
| RENT3 | 0.17 | 0.49 | 0.60 | 0.65 | 0.46 | 0.61 | 0.52 | 0.42 | 0.16 | 0.34 | 1.00 | |
| Bitcoin | 0.10 | 0.28 | 0.06 | 0.09 | 0.09 | 0.19 | 0.16 | 0.17 | 0.23 | 0.07 | 0.20 | 1.00 |

In both periods analyzed, Bitcoin showed low correlation with the selected stocks, with values ranging as follows:
• 5-year horizon: from 0.08 with ELET3 to 0.37 with JBSS3 and PRIO3;
• 10-year horizon: from 0.06 with BBAS3 to 0.28 with PRIO3.

These levels are considered low, indicating that Bitcoin's price movements do not closely follow those of the selected stocks. In other words, when stocks rise or fall, Bitcoin does not necessarily move in the same direction. This characteristic helps stabilize the investment portfolio, since sharp fluctuations in other assets may be partially offset by Bitcoin, thereby improving the portfolio's risk-return profile.

Another indicator that contributed to understanding the relationships among assets was covariance. Tables 6 and 7 present the covariance values.

**Table 6**

*Covariance matrix of asset returns (5-year horizon)*

| | JBSS3 | PRIO3 | BBAS3 | BBDC4 | ELET3 | EQTL3 | SBSP3 | WEGE3 | EMBR3 | ABEV3 | RENT3 | Bitcoin |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| JBSS3 | 0.0078 | | | | | | | | | | | |
| PRIO3 | 0.0055 | 0.0338 | | | | | | | | | | |
| BBAS3 | 0.0013 | 0.0107 | 0.0103 | | | | | | | | | |
| BBDC4 | 0.0020 | 0.0094 | 0.0077 | 0.0111 | | | | | | | | |
| ELET3 | 0.0019 | 0.0116 | 0.0057 | 0.0061 | 0.0111 | | | | | | | |
| EQTL3 | 0.0017 | 0.0093 | 0.0052 | 0.0058 | 0.0051 | 0.0069 | | | | | | |
| SBSP3 | 0.0017 | 0.0109 | 0.0053 | 0.0053 | 0.0062 | 0.0063 | 0.0111 | | | | | |
| WEGE3 | 0.0008 | 0.0062 | 0.0013 | 0.0030 | 0.0037 | 0.0041 | 0.0043 | 0.0100 | | | | |
| EMBR3 | 0.0040 | 0.0123 | 0.0049 | 0.0051 | 0.0049 | 0.0039 | 0.0054 | 0.0026 | 0.0222 | | | |
| ABEV3 | 0.0013 | 0.0059 | 0.0040 | 0.0049 | 0.0034 | 0.0034 | 0.0037 | 0.0014 | 0.0042 | 0.0054 | | |
| RENT3 | 0.0018 | 0.0145 | 0.0085 | 0.0089 | 0.0080 | 0.0072 | 0.0084 | 0.0068 | 0.0050 | 0.0035 | 0.0167 | |
| Bitcoin | 0.0061 | 0.0130 | 0.0038 | 0.0030 | 0.0015 | 0.0043 | 0.0030 | 0.0046 | 0.0094 | 0.0029 | 0.0064 | 0.0357 |

**Table 7**

*Covariance matrix of asset returns (10-year horizon)*

| | JBSS3 | PRIO3 | BBAS3 | BBDC4 | ELET3 | EQTL3 | SBSP3 | WEGE3 | EMBR3 | ABEV3 | RENT3 | Bitcoin |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| JBSS3 | 0.0108 | | | | | | | | | | | |
| PRIO3 | 0.0024 | 0.0371 | | | | | | | | | | |
| BBAS3 | 0.0007 | 0.0110 | 0.0128 | | | | | | | | | |
| BBDC4 | 0.0016 | 0.0085 | 0.0087 | 0.0098 | | | | | | | | |
| ELET3 | 0.0033 | 0.0072 | 0.0091 | 0.0076 | 0.0198 | | | | | | | |
| EQTL3 | 0.0015 | 0.0058 | 0.0048 | 0.0048 | 0.0055 | 0.0055 | | | | | | |
| SBSP3 | 0.0013 | 0.0074 | 0.0054 | 0.0050 | 0.0071 | 0.0054 | 0.0107 | | | | | |
| WEGE3 | 0.0005 | 0.0045 | 0.0015 | 0.0019 | 0.0022 | 0.0024 | 0.0027 | 0.0073 | | | | |
| EMBR3 | 0.0023 | 0.0056 | 0.0021 | 0.0021 | 0.0006 | 0.0017 | 0.0028 | 0.0009 | 0.0150 | | | |
| ABEV3 | 0.0012 | 0.0031 | 0.0026 | 0.0032 | 0.0024 | 0.0022 | 0.0027 | 0.0017 | 0.0018 | 0.0043 | | |
| RENT3 | 0.0019 | 0.0105 | 0.0076 | 0.0072 | 0.0073 | 0.0051 | 0.0060 | 0.0040 | 0.0022 | 0.0025 | 0.0125 | |
| Bitcoin | 0.0021 | 0.0108 | 0.0014 | 0.0018 | 0.0025 | 0.0029 | 0.0033 | 0.0029 | 0.0057 | 0.0009 | 0.0044 | 0.0406 |

To analyze the covariance matrix, it is essential to understand how assets behave jointly. As with correlation, positive covariances indicate movement in the same direction, whereas negative covariances indicate movement in opposite directions. However, covariance is also affected by the volatility of the assets. Because Bitcoin is highly volatile, it may present high covariance with certain assets even when correlations are low.

Among the assets, JBSS3 stands out for presenting the lowest covariances relative to other stocks in both periods. As a food commodity exporter, it follows a different dynamic from other companies, particularly those in the financial and consumer sectors, resulting in lower-intensity covariances. Conversely, PRIO3 concentrated the highest values in both periods, reflecting its high volatility.

Regarding Bitcoin, the covariance values showed substantial variation, ranging from 0.0015 with ELET3 to 0.0130 with PRIO3 over the 5-year period, and from 0.009 with ABEV3 to 0.0108 with PRIO3 over the 10-year period. This behavior contributes positively to the risk-return relationship when correlation is low and covariance is high, supporting an improvement in the Sharpe ratio. However, when both correlation and covariance remain high, the level of risk increases without generating proportional gains.

**Composition of Optimized Portfolios (without Bitcoin)**

Understanding the relationships among assets and their behavior is essential for constructing efficient investment portfolios. Analyzing these indicators provides a deeper understanding of the risk-return tradeoff, which lies at the core of Modern Portfolio Theory (MPT). Based on a detailed examination of volatility, correlation, and covariance, it becomes possible to advance to the practical stage of asset allocation. To obtain the composition of the portfolios that minimize risk and maximize the Sharpe ratio (SR), the Microsoft Excel Solver tool was employed. For the calculation of the SR, average SELIC rates of 8.875% and 9.45625% were used for the five- and ten-year periods, respectively. The results, excluding Bitcoin, for both investment horizons are presented in Tables 8 and 9.

**Table 8**

*Portfolio allocations without bitcoin (5-year horizon)*

| Assets | Minimum-risk portfolio | Optimal portfolio |
|---|---|---|
| JBSS3 | 30.0% | 0.0% |
| PRIO3 | 0.0% | 25.9% |
| BBAS3 | 8.6% | 0.0% |
| BBDC4 | 0.0% | 0.0% |
| ELET3 | 0.0% | 0.0% |
| EQTL3 | 0.0% | 0.0% |
| SBSP3 | 0.0% | 0.0% |
| WEGE3 | 22.4% | 62.2% |
| EMBR3 | 0.0% | 11.9% |
| ABEV3 | 39.0% | 0.0% |
| RENT3 | 0.0% | 0.0% |

**Table 9**

*Portfolio allocations without bitcoin (10-year horizon)*

| Assets | Minimum-risk portfolio | Optimal portfolio |
|---|---|---|
| JBSS3 | 15.8% | 11.1% |
| PRIO3 | 0.0% | 29.1% |
| BBAS3 | 3.2% | 0.0% |
| BBDC4 | 0.0% | 0.0% |
| ELET3 | 0.0% | 0.9% |
| EQTL3 | 15.8% | 0.9% |
| SBSP3 | 0.0% | 0.0% |
| WEGE3 | 20.5% | 58.0% |
| EMBR3 | 7.5% | 0.0% |
| ABEV3 | 37.2% | 0.0% |
| RENT3 | 0.0% | 0.0% |

Regarding the minimum-risk portfolio, the five-year horizon highlights the importance of JBSS3, WEGE3, and ABEV3. This result is consistent with the indicators previously analyzed, as JBSS3 and ABEV3 exhibited the lowest covariance values (0.0078 and 0.0054, respectively), combined with low correlations, contributing to a reduction in portfolio volatility. Although WEGE3 displayed somewhat higher volatility (10.01%), it maintained low correlations with the remaining assets, reinforcing its role in reducing overall portfolio risk.

Over the 10-year period, the minimum-risk portfolio incorporated EQTL3. Its inclusion is justified by its low volatility (7.42%) and low covariance with JBSS3, WEGE3, and ABEV3 (0.0015, 0.0022, and 0.0024, respectively).

In the optimal portfolios, PRIO3 and WEGE3 played prominent roles in both periods. The presence of PRIO3 is directly justified by its high returns (3.01% and 3.72% over the 5- and 10-year periods, respectively). Despite its substantial volatility (19.27%), its returns more than compensate for the additional risk incurred. In contrast, WEGE3 delivered more moderate returns (1.97% in both periods), while contributing to risk stabilization and maximizing the Sharpe ratio.

Table 10 presents the results for the five-year horizon. The portfolio exhibits the lowest volatility level observed in the study, reflecting the inclusion of more defensive assets. The expected return mirrors the emphasis on portfolio stability, resulting in more moderate performance. In contrast, the optimal portfolio demonstrates a substantial increase in expected return due to the inclusion of more aggressive assets. Risk also increased, although to a lesser extent, reflecting the maximization of the Sharpe ratio, which seeks the most favorable risk-return tradeoff.

**Table 10**

*Risk and return characteristics of portfolios without Bitcoin (5-year horizon)*

| | **Minimum-risk portfolio** | **Optimal portfolio** |
|---|---|---|
| Return | 0.6% | 2.2% |
| Risk | 5.6% | 9.8% |

The results of the 10-year simulation are shown in Table 11. The minimum-risk and optimal portfolios yielded results similar to those of the 5-year simulation.

**Table 11**

*Risk and return characteristics of portfolios without Bitcoin (10-year horizon)*

| | **Minimum-risk portfolio** | **Optimal portfolio** |
|---|---|---|
| Return | 0.9% | 2.4% |
| Risk | 5.2% | 8.7% |

## Composition of Optimized Portfolios (with Bitcoin)

Bitcoin was subsequently introduced into the portfolio optimization process, again considering both the 5- and 10-year horizons. Tables 12 and 13 present the resulting allocations, allowing changes in asset weights to be observed.

**Table 12**

*Portfolio allocations with Bitcoin (5-year horizon)*

| **Assets** | **Minimum-risk portfolio** | **Optimal portfolio** |
|---|---|---|
| JBSS3 | 30.0% | 0.0% |
| PRIO3 | 0.0% | 12.9% |
| BBAS3 | 8.6% | 0.0% |
| BBDC4 | 0.0% | 0.0% |
| ELET3 | 0.0% | 0.0% |
| EQTL3 | 0.0% | 0.0% |
| SBSP3 | 0.0% | 0.0% |
| WEGE3 | 22.4% | 41.3% |
| EMBR3 | 0.0% | 0.0% |
| ABEV3 | 39.0% | 0.0% |
| RENT3 | 0.0% | 0.0% |
| Bitcoin | 0.0% | 45.8% |

**Table 13**

*Portfolio allocations with Bitcoin (10-year horizon)*

| Assets | Minimum-risk portfolio | Optimal portfolio |
|---|---|---|
| JBSS3 | 15,6% | 5,8% |
| PRIO3 | 0,0% | 17,4% |
| BBAS3 | 3,4% | 0,0% |
| BBDC4 | 0,0% | 0,0% |
| ELET3 | 0,0% | 0,8% |
| EQTL3 | 15,1% | 0,8% |
| SBSP3 | 0,0% | 0,0% |
| WEGE3 | 20,0% | 43,3% |
| EMBR3 | 7,0% | 0,0% |
| ABEV3 | 37,4% | 0,0% |
| RENT3 | 0,0% | 0,0% |
| Bitcoin | 1,4% | 31,9% |

Even with the inclusion of Bitcoin, it did not enter the minimum-risk portfolio over the 5-year period. This outcome is primarily explained by Bitcoin's high volatility (18.9%) relative to the other assets. In the 10-year horizon, however, Bitcoin was incorporated into the minimum-risk portfolio, albeit with a small allocation (1.4%). This result can be explained by its low correlations and covariances with the remaining assets over the long term, enabling Bitcoin to provide a marginal diversification benefit. Its presence demonstrates that high-risk assets may, under certain circumstances, contribute to reducing overall portfolio risk when their statistical relationships with other assets are sufficiently weak.

In the optimal portfolios, Bitcoin emerged as a prominent asset alongside PRIO3 and WEGE3 in both periods. Its inclusion reflects its high average return (exceeding 4% per month in both horizons), which, despite its elevated volatility, contributes to maximizing the Sharpe ratio when combined with more balanced assets. Within this combination, WEGE3 continues to serve as a stabilizing component, while PRIO3 contributes through its strong return performance. These findings reinforce the importance of low-correlation assets in the construction of diversified portfolios and highlight Bitcoin as a viable instrument for enhancing the risk-return profile. The final results are presented in Tables 14 and 15.

**Table 14**

*Risk and return characteristics of portfolios with Bitcoin (5-year horizon)*

| | Minimum-risk portfolio | Optimal portfolio |
|---|---|---|
| Return | 0.6% | 3.2% |
| Risk | 5.6% | 11.7% |

**Table 15**

*Risk and return characteristics of portfolios with Bitcoin (10-year horizon)*

| | Minimum-risk portfolio | Optimal portfolio |
|---|---|---|
| Return | 1.0% | 3.1% |
| Risk | 5.2% | 9.8% |

In the short term, Bitcoin's absence from the minimum-risk portfolio resulted in risk and return levels identical to those observed in the previous simulation. The optimal portfolio, however, achieved substantial improvements. Although overall risk increased, the increase in expected return was proportionally greater. Over the 10-year horizon, Bitcoin's modest allocation slightly increased expected return without materially affecting risk. The optimal portfolio produced returns similar to those observed in the shorter horizon but with lower volatility.

## Efficient Frontier and Portfolio Simulation

After calculating the minimum-risk and optimal portfolios and completing the preceding analyses, the study proceeded to the scenario simulation and efficient frontier construction stage. The objective was to evaluate how different combinations of the portfolios affected the risk-return relationship, thereby identifying the most efficient allocation points.

The simulation tables were generated by combining the previously calculated portfolios in different proportions across both investment horizons, ranging from 0% to 100% allocation in each portfolio. The “% minimum risk” column indicates the percentage allocated to the lowest-volatility portfolio, while the “% optimal” column represents the percentage allocated to the portfolio that maximizes the Sharpe ratio. The subsequent columns display the resulting asset allocations from each portfolio combination, calculated as the weighted sum of the asset allocations in the minimum-risk and optimal portfolios (Tables 9 and 10). Finally, the “Return” and “Risk” columns present the projected outcomes for each simulated scenario.

The simulations enabled the construction of efficient frontier curves for both the 5- and 10-year horizons, visually highlighting differences in portfolio performance. Figures 3 and 4 present these results. The shapes obtained from the simulations follow the theoretical pattern predicted by Markowitz, whereby the frontier shifts outward as the risk-return relationship is optimized. This consistency between empirical findings and theoretical expectations supports the practical applicability of MPT, even in contemporary contexts involving unconventional digital assets.

For the 5-year horizon, the inclusion of Bitcoin shifted the efficient frontier upward, increasing expected returns across most levels of risk. This result suggests that, despite its high volatility, Bitcoin contributes positively to risk-adjusted performance when combined with assets that help balance overall portfolio risk.

A similar effect was observed over the 10-year horizon, although more pronounced. This finding suggests that longer investment horizons tend to benefit more substantially from Bitcoin inclusion. Such an effect may be attributed to Bitcoin’s strong long-term return profile and its low correlation with traditional assets.

**Figure 3**

Efficient frontier – 5-year horizon

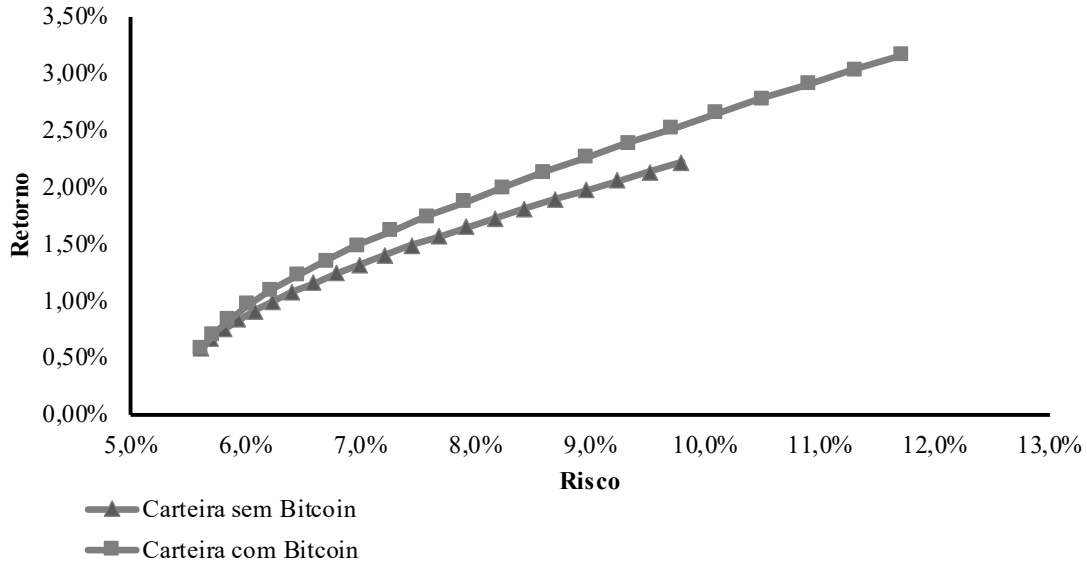
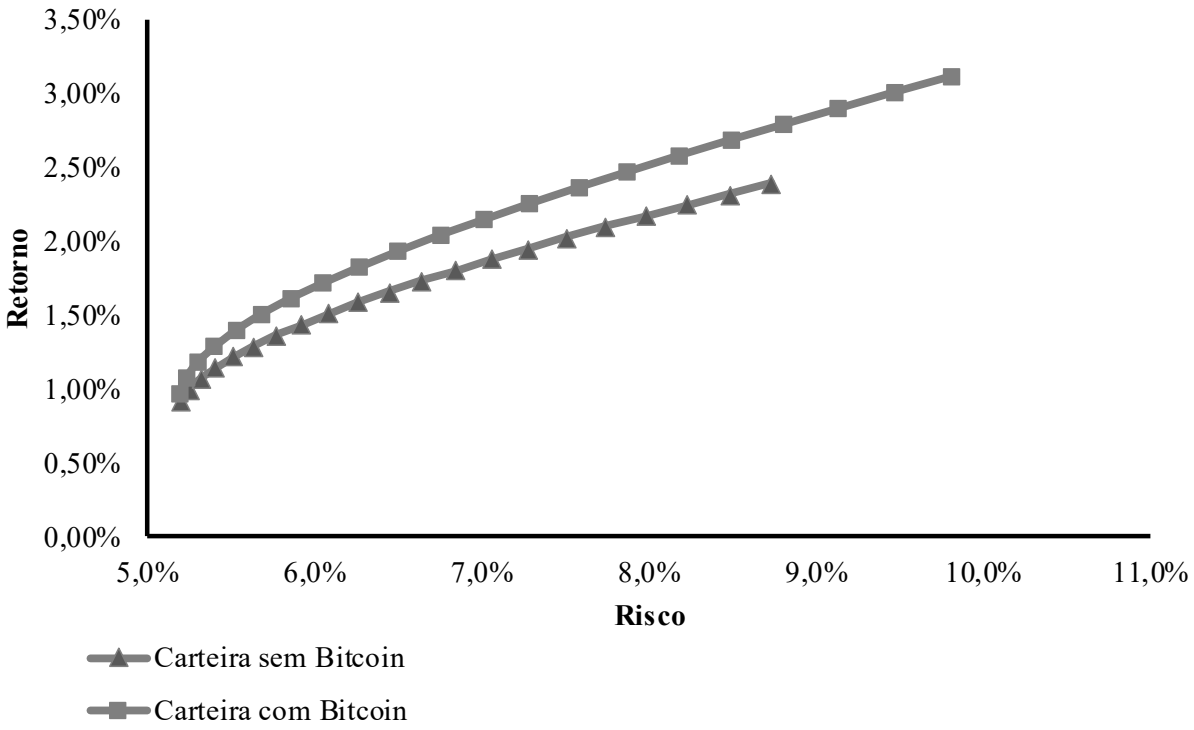


**Figure 4**

Efficient frontier – 10-year horizon




To assess the robustness of the main findings, two complementary exercises were conducted, the results of which are summarized in Table 16.

First, an out-of-sample test was performed by applying the optimal portfolio weights estimated from the 2014–2019 period to the realized returns observed between 2020 and 2024. The portfolio including Bitcoin maintained a higher Sharpe ratio than the portfolio composed exclusively of traditional assets, indicating that the diversification benefits are not limited to the estimation period.

Second, a sensitivity analysis was conducted by imposing increasingly restrictive constraints on the maximum allocation to Bitcoin (45.79% to 10%). Even under severe allocation constraints, the Sharpe ratio consistently remained higher than that of the portfolio without Bitcoin across both investment horizons. This finding demonstrates that efficiency gains can be achieved even with moderate Bitcoin exposures (10–20%), which are more realistic for most Brazilian investor profiles.

**Table 16**

*Portfolio sensitivity analysis: maximum bitcoin allocation constraints and their impact on the Sharpe Ratio (5- and 10-year horizons)*

5-year period

| Scenario | Bitcoin allocation (%) | Sharpe ratio |
|---|---|---|
| Optimal portfolio | 45.79% | 20.92% |
| 1 | 35.00% | 20.68% |
| 2 | 30.00% | 20.37% |
| 3 | 25.00% | 19.92% |
| 4 | 20.00% | 19.31% |

10-year period

| Scenario | Bitcoin allocation (%) | Sharpe ratio |
|---|---|---|
| Optimal portfolio | 31.88% | 24.01% |
| 1 | 25.00% | 23.85% |
| 2 | 20.00% | 23.47% |
| 3 | 15.00% | 22.78% |
| 4 | 10.00% | 21.70% |

## Comparison with market indices

Finally, the optimized portfolios were compared with the following market indices:

• IBOV (Ibovespa): the principal Brazilian stock market index, composed of the most liquid and representative stocks;
• IDIV (Dividend Index): measures the performance of major dividend-paying companies;
• ISE (Corporate Sustainability Index): tracks the performance of companies committed to ESG practices;
• IEE (Electric Energy Index): follows the average performance of companies in the electric energy sector;
• IMAT (Basic Materials Index): measures the performance of companies operating in sectors such as pulp and paper, mining, and steel production.

The return and risk characteristics of each index are presented in Tables 17 and 18.

**Table 17**

*Risk and return characteristics of market indices (5-year horizon)*

| Index | Return | Risk |
|---|---|---|
| IBOV | 0.1% | 7.3% |
| IDIV | 0.4% | 6.2% |
| ISE | -0.5% | 7.6% |
| IEE | 0.0% | 6.0% |
| IMAT | 0.7% | 7.5% |

**Table 18**

*Risk and return characteristics of market indices (10-year horizon)*

| **Index** | **Return** | **Risk** |
|---|---|---|
| IBOV | 0.7% | 6.6% |
| IDIV | 1.0% | 6.6% |
| ISE | 0.2% | 6.4% |
| IEE | 0.9% | 6.0% |
| IMAT | 1.2% | 7.6% |

Comparing the indices with the efficient frontiers reveals that, in both periods, the minimum-risk portfolio exhibited lower volatility (5.0%-5.6%) than all benchmark indices (6.0%-7.6%). In terms of return, however, the minimum-risk portfolio produced relatively modest results (0.58%-0.97%), remaining below the performance of the IMAT index.

The optimal portfolio, in contrast, generated substantially higher returns, albeit with greater volatility. Expected returns exceeded 2% per month without Bitcoin and surpassed 3% per month when Bitcoin was included. Figures 5 and 6 provide graphical comparisons between the efficient frontiers and the market indices.

**Figure 5**

*Efficient frontier versus market indices – 5-year horizon*

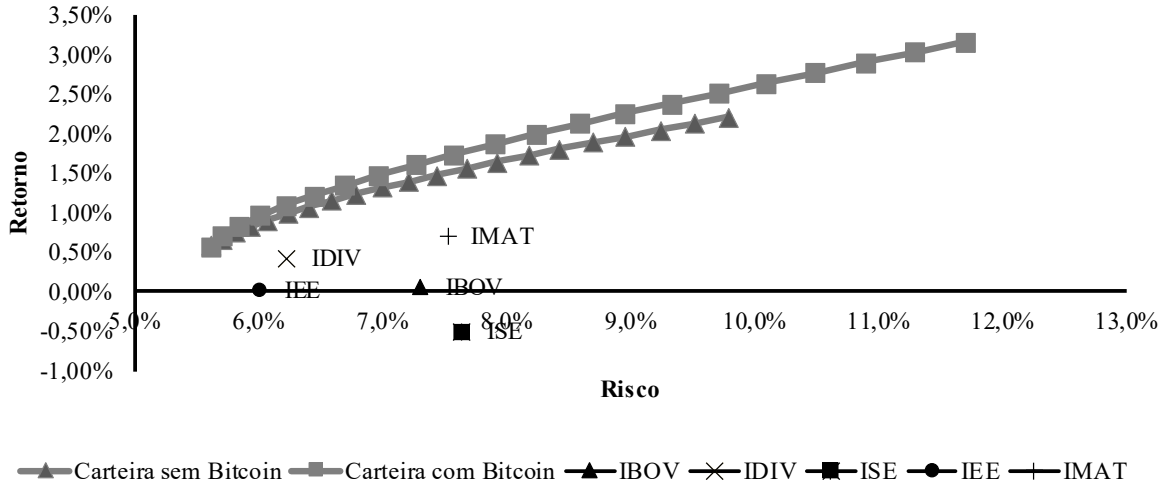
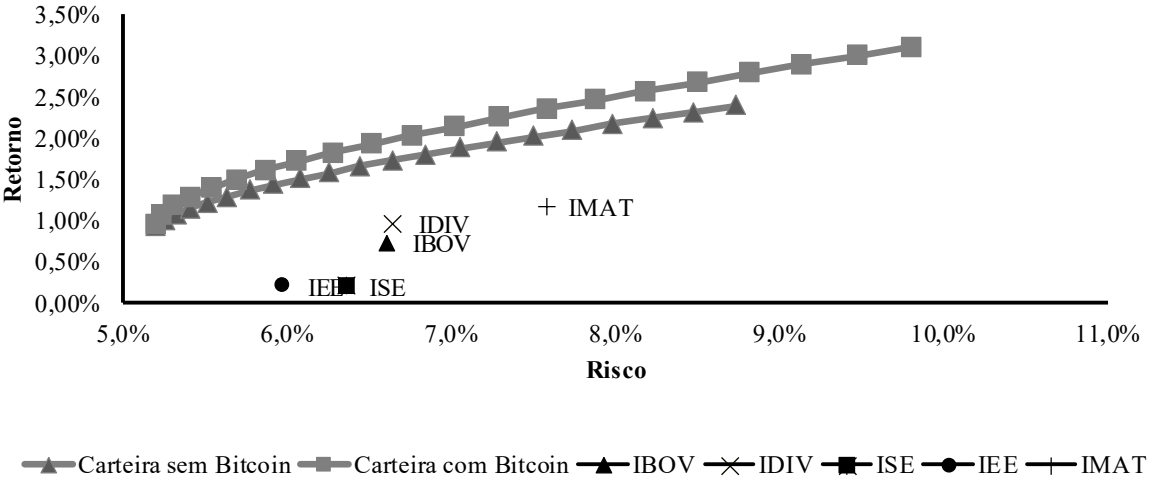



**Figure 6**

*Efficient frontier versus market indices – 10-year horizon*




Although certain indices exhibited superior risk and/or return metrics individually, they remained below the efficient frontier, underscoring the importance of portfolio optimization and the benefits of achieving a more favorable risk-return tradeoff through strategic asset allocation.

## Conclusions

The findings of this study confirm that the inclusion of Bitcoin in portfolios composed of traditional assets improves the risk-return tradeoff, consistent with the principles of MPT. Despite Bitcoin's high volatility, its low correlation with B3-listed assets shifts the efficient frontier upward, enabling more favorable risk-return combinations, as suggested by Neves and Silva (2017) and Kohatsu et al. (2023).

Bitcoin's decentralized nature and relative independence from conventional economic cycles reinforce its role as a diversification instrument, particularly in optimal portfolios containing assets such as WEGE3 and PRIO3, thereby maximizing the Sharpe ratio (Oliveira et al., 2020). Over both 5- and 10-year horizons, its low correlation and strong long-term return profile enhance its positive contribution to portfolio performance, consistent with the findings of Kohatsu et al. (2023) and Pakes et al. (2022).

### Contributions to the Literature

This study contributes to the financial literature by providing empirical evidence supporting the applicability of Modern Portfolio Theory in the era of digital assets. The results indicate that the inclusion of Bitcoin can enhance portfolio efficiency by reducing unsystematic risk, corroborating the findings of Neves and Silva (2017), Kohatsu et al. (2023), and Batista and Alves (2021). The observed upward shift of the efficient frontier resulting from the inclusion of low-correlation assets demonstrates the continued validity of MPT assumptions even in environments characterized by significant financial innovation.

Traditional portfolio literature has largely focused on conventional asset classes such as equities, fixed income securities, and real estate investment funds (Gonçalves et al., 2024; Zanini & Figueiredo, 2005), often overlooking cryptoassets. By incorporating Bitcoin, the present study addresses an important gap highlighted by Pakes et al. (2022), demonstrating that a controlled allocation to digital assets is not only feasible but may also be advantageous from a risk-return optimization perspective.

Another important contribution lies in the empirical evaluation of Bitcoin's behavior across two distinct investment horizons. By showing that its positive impact on portfolio efficiency becomes more pronounced over longer horizons, the study reinforces the arguments of Ferreira-Schenk and Dickason-Koekemoer (2023) regarding the consistency of digital asset returns during periods of macroeconomic instability, such as the COVID-19 pandemic.

### Managerial Contributions

From a managerial perspective, the study provides practical insights for portfolio managers, financial planners, and institutional investors. First, by demonstrating that the inclusion of Bitcoin in equity portfolios improves allocation efficiency, the findings offer a compelling rationale for incorporating cryptoassets into diversified investment strategies.

Consistent with previous studies (Batista & Alves, 2021; Oliveira et al., 2020), the results indicate that Bitcoin can function as an effective diversification instrument due to its low correlation with Brazilian equities, thereby reducing unsystematic risk. This effect becomes even more significant over longer horizons, when volatility-smoothing benefits become increasingly evident, as suggested by Kohatsu et al. (2023).

Another relevant managerial implication concerns the possibility of constructing customized portfolios. The simulation of minimum-risk and optimal portfolios provides managers with greater flexibility to accommodate a wide range of investor risk profiles, from conservative to aggressive. The quantitative framework presented in this study may serve as a practical guide for informed decision-making, aligning return objectives with risk tolerance.

### Limitations and Directions for Future Research

Despite its findings, this study presents several limitations that should be acknowledged. The first relates to the assumptions underlying Modern Portfolio Theory, which presumes normally distributed returns and treats variance as a complete measure of risk. The statistical analyses revealed substantial skewness and kurtosis among the assets, indicating departures from normality. Consequently, extreme events may occur more frequently than predicted by the model.

A second limitation concerns the stability of the estimated parameters, particularly the correlation and covariance matrices. These parameters vary over time and are sensitive to the period selected for analysis. As a result, portfolio weights reflect historical behavior and do not guarantee optimality under future market conditions.

Importantly, the study relied on historical data and average risk-free rates for each period. Alternative time windows, data frequencies, or estimation methods could potentially yield different results. Finally, Bitcoin remains a relatively young and highly volatile asset, increasing uncertainty and potentially affecting the stability of portfolios that include it.

Therefore, although Bitcoin provided diversification benefits during the periods analyzed, there is no guarantee that similar performance will be observed in the future. Nevertheless, the out-of-sample tests and sensitivity analyses demonstrated that the results are robust and that the inclusion of Bitcoin improves portfolio efficiency across different market environments.

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